CSE 3400 - Introduction to Computer & Network Security (aka: Introduction to Cybersecurity)

Lecture 11
Public Key Cryptography– Part II

Ghada Almashaqbeh
UConn

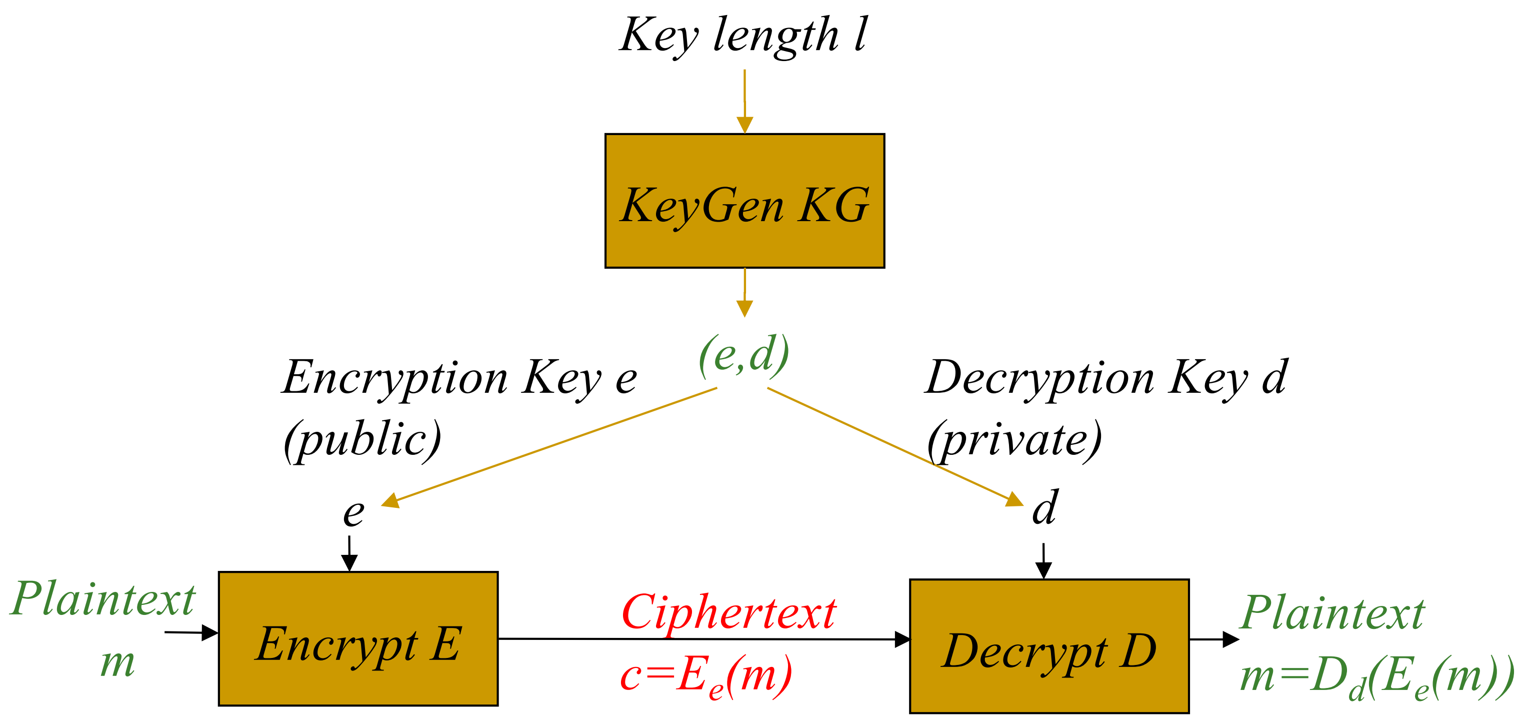
From Textbook Slides by Prof. Amir Herzberg
UConn

Outline

- Public key encryption.
- Digital signatures.
- PKI

Public Key Encryption

Public Key Encryption



Public Key Encryption IND-CPA Security

$T^{IND-CPA}_{\mathcal{A},\langle KG,E,D\rangle}(b,n)$ {

$(e,d) \overset{\$}{\leftarrow} KG(1^n)$

$(m_0,m_1) \leftarrow \mathcal{A}(\text{‘Choose’}, e)$ s.t. $|m_0| = |m_1|$

$c^* \leftarrow E_e(m_b)$

$b^* = \mathcal{A}(\text{‘Guess’}, (c^*, e))$

Return b^*

}

Definition 2.10 (PKC IND-CPA). *Let $\langle KG, E, D\rangle$ be a public-key cryptosystem. We say that $\langle KG, E, D\rangle$ is* IND-CPA*, if every efficient adversary $\mathcal{A} \in PPT$ has negligible advantage $\varepsilon^{IND-CPA}_{<KG,E,D>,\mathcal{A}}(n) \in NEGL(n)$, where:*

$$\varepsilon^{IND-CPA}_{\langle KG,E,D\rangle,\mathcal{A}}(n) \equiv \Pr\left[T^{IND-CPA}_{\mathcal{A},\langle KG,E,D\rangle}(1,n) = 1\right] - \Pr\left[T^{IND-CPA}_{\mathcal{A},\langle KG,E,D\rangle}(0,n) = 1\right] \tag{2.35}$$

Where the probability is over the random coin tosses in IND-CPA (including of $\mathcal{A}$ and E).

Discrete Log-based Encryption

- We will explore two flavors:
  - An adaptation of DH key exchange protocol to perform encryption.
  - ElGamal encryption scheme.

Turning [DH] to Public Key Cryptosystem

- Solves dependency on DDH assumption; secure under the (weaker) CDH assumption.
- To encrypt message m to Alice:
  - Bob selects random b
  - Sends: $g^b \bmod p$, $m \oplus h((e_A)^b) = m \oplus h(g^{b \cdot d_A} \bmod p)$
  - Secure if $h(g^{b \cdot d_A} \bmod p)$ is pseudo-random

Alice

$e_A = g^{d_A} \bmod p$ →

← $g^b \bmod p$, $m \oplus h(g^{b \cdot d_A} \bmod p)$

Bob

ElGamal Public Key Encyption

- Variant of [DH] PKC: Encrypt by multiplication, not XOR
- To encrypt message m to Alice, whose public key is $e_A = g^{d_A} \; mod \; p$:
  - Bob selects random b
  - Sends: $g^b \; mod \; p \; , \; m*(e_A)^b = m*g^{b \cdot d_A} \; mod \; p$

Alice

$e_A = g^{d_A} \; mod \; p$

$(g^b \; mod \; p \; , \; (m* e_A^b) \; mod \; p)$

Bob

Select random b

ElGamal Public Key Encryption

- Encryption:

$$E^{EG}_{e_A}(m) \leftarrow \left\{ \left(g^b \mod p\ ,\ m \cdot e_A^b \mod p\right) | b \xleftarrow{\$} [2, p-1] \right\}$$

- Decryption:

$$D_{d_A}(x, y) = x^{-d_A} \cdot y \mod p$$

- Correctness:

$$\begin{aligned} D_{d_A}(g^b \mod p\ ,\ m \cdot\ e_A^b \mod p) &= \\ &= \left[\left(g^b \mod p\right)^{-d_A} \cdot \left(m \cdot \left(g^{d_A}\right)^b \mod p \right) \right] \mod p \\ &= \left[g^{-b \cdot d_A} \cdot m \cdot g^{b \cdot d_A} \right] \mod p \\ &= m \end{aligned}$$

ElGamal Public Key Cryptosystem

- Problem: $g^{b \cdot d_A} \ \ mod\, p$ may leak bit(s)…
- \`Classical' DH solution: securely derive a key: $h\left(g^{a_i b_i} mod\ p\right)$
- El-Gamal's solution: use a group where DDH believed to hold
  - Note: message must be encoded as member of the group!
  - So why use it? Some special properties…

ElGamal PKC: homomorphism

- Multiplying two ciphertexts produces a ciphertext of the multiplication of the two plaintexts.
- Given two ciphertexts:
  - $E_{e_A}(m_1) = (x_1, y_1) = (g^{b_1} \bmod p, m_1 * g^{b_1 \cdot d_A} \bmod p)$
  - $E_{e_A}(m_2) = (x_2, y_2) = (g^{b_2} \bmod p, m_2 * g^{b_2 \cdot d_A} \bmod p)$
- $Mult\big((x_1, y_1), (x_2, y_2)\big) \equiv (x_1 x_2, y_1 y_2)$
- Homomorphism:
- $= (g^{b_1+b_2} \bmod p, m_1 \cdot m_2 * g^{(b_1+b_2) \cdot d_A} \bmod p) =$

  $= E_{e_A}(m_1 \cdot m_2)$
- ➔ compute $E_{e_A}(m_1 \cdot m_2)$ from $E_{e_A}(m_1)$, $E_{e_A}(m_1)$

RSA Public Key Encryption

2002 Turing Award

- First proposed – and still widely used
- Not really covered in this course – take crypto!
- Select two large primes p,q ; let $n=pq$
- Select prime e (public key: $<n,e>$)
  - Or co-prime with $\Phi(n) = (p-1)(q-1)$
- Let private key be $d=e^{-1} \bmod \Phi(n)$ (i.e., $ed=1 \bmod \Phi(n)$)
- Encryption: $RSA.E_{e,n}(m)=m^e \bmod n$
- Decryption: $RSA.D_{d,n}(c)=c^d \bmod n$
- Correctness: $D_{d,n}(E_{e,n}(m)) = (m^e)^d = m^{ed} = m \bmod n$
  - Intuitively: $ed=1 \bmod \Phi(n) \Rightarrow m^{ed} = m \bmod n$
- But why? Remember Euler's theorem.

RSA Public Key Cryptosystem

- Correctness: $D_{d,n}(E_{e,n}(m))= m^{ed}\ mod\ n$
- $m^{ed}=m^{ed}= m^{1+l\ \Phi(n)} =m\ m^{l\ \Phi(n)} =m\ (m^{\Phi(n)}\ )^{l}$
- $m^{ed}\ mod\ n =m\ (m^{\Phi(n)}\ mod\ n\ )^{l}\ mod\ n$
- Eulers'Theorem: $m^{\Phi(n)}\ mod\ n=1\ mod\ n$
- $\rightarrow D_{d,n}(E_{e,n}(m))= m^{ed}\ mod\ n=m\ 1^{l}\ mod\ n =m$
- Comments:
  - $m<n \rightarrow m= m\ mod\ n$
  - Eulers' Theorem holds (only) if m, n are co-primes
  - If not co-primes? Use Chinese Reminder Theorem
    - A nice, not very complex argument
    - But: beyond our scope – take Crypto!

The RSA Problem and Assumption

- RSA problem: Find m , given (n,e) and ‘ciphertext’ value $c=m^e \bmod n$
- RSA assumption: if (n,e) are chosen \`correctly’, then the RSA problem is \`hard’
  - I.e., no efficient algorithm can find m with non-negligible probability
  - For \`large’ n and $m \overset{\$}{\leftarrow} \{1, \dots, n\}$
- RSA and factoring
  - Factoring alg ➔ alg to ‘break’ RSA
  - Algorithm to find RSA private key ➔ factoring alg
  - But: RSA-breaking may <u>*not*</u> allow factoring

RSA PKC Security

- It is a deterministic encryption scheme → cannot IND-CPA secure.
- RSA assumption does not rule out exposure of partial information about the plaintext.
- It is not CCA secure.

A solution: apply a random padding to the plaintext then encryption using RSA.

Padding RSA

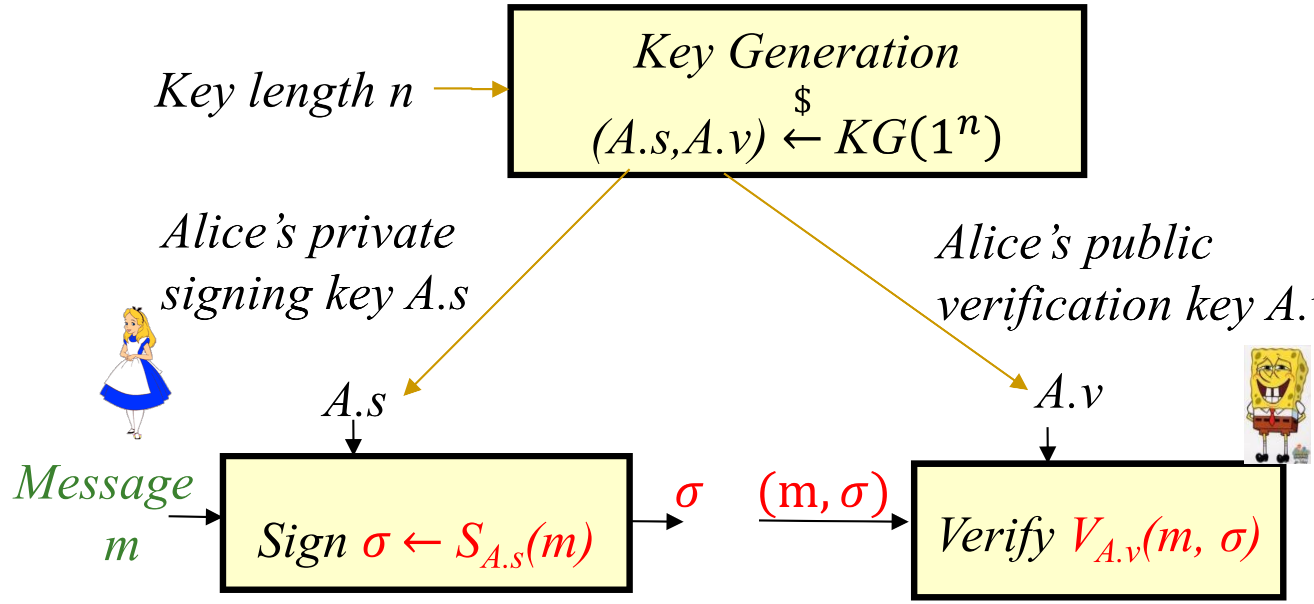
- Pad and Unpad functions: $m = Unpad(Pad(m;r))$
  - Encryption with padding: $c = [Pad(m,r)]^e \bmod n,$
  - Decryption with unpad: $m = Unpad(c^d \bmod n)$
- Required to…
  - Add randomization
    - Prevent detection of repeating plaintext
  - Prevent ‘related message’ attack (to allow use of tiny e)
  - Detect, prevent (some) chosen-ciphertext attacks
- Early paddings schemes subject to CCA attacks
  - Even ‘Feedback-only CCA’ (aware of unpad failure)

How does Bob know Alice's public key?

- Depends on threat model…
  - Passive (`eavesdropping`) adversary: just send it
  - Man-in-the-Middle (MITM): **authenticate**
- Authenticate – how?
  - MAC: requires shared secret key
  - **Public key signature scheme**: authenticate using public key
  - Certificate: public key of entity – **signed** by **certificate authority (CA)**
    - **This comes under what is called Public Key Infrastructure (PKI)**

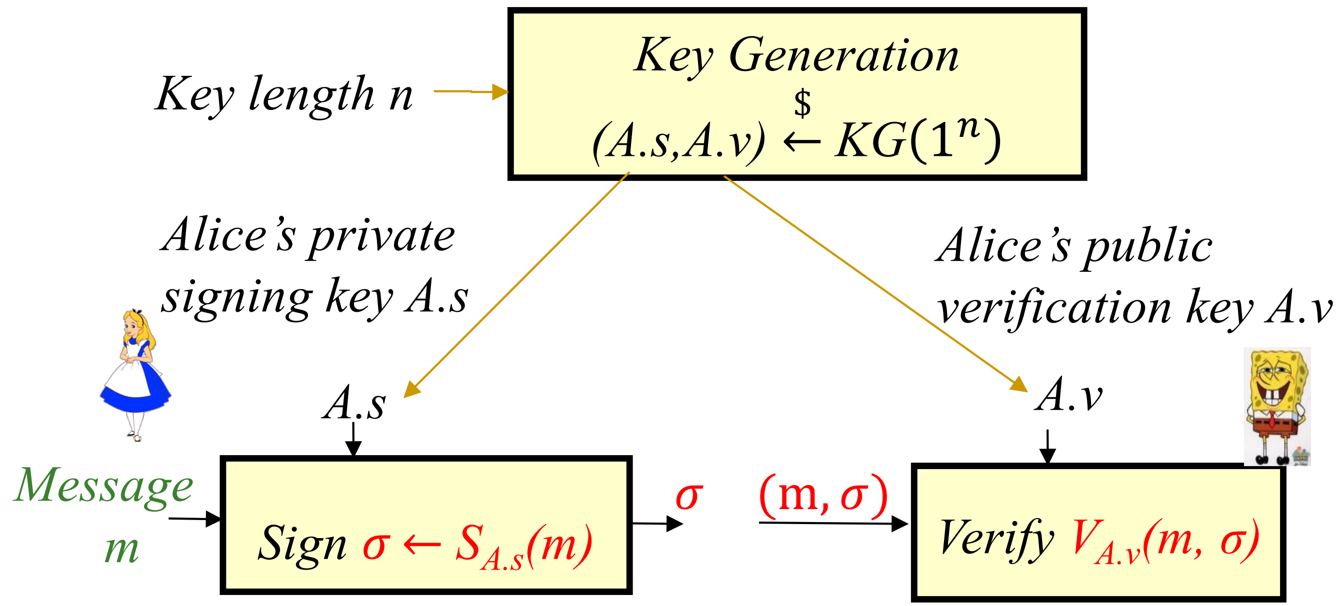
Digital Signature

Public Key Digital Signatures



- Sign using a private, secret signature key ($A.s$ for Alice)
- Validate using a <u>public</u> key ($A.v$ for Alice)
- Everybody can validate signatures at any time
  - Provides authentication, integrity **<u>and</u>** evidence / non-repudiation
  - MAC: 'just' authentication+integrity, no evidence, can repudiate

Digital Signatures Security: Unforgeability



- Unforgeability: given v, attacker should be unable to find **any** 'valid' (m, σ), i.e., $V_v(m, \sigma)=OK$
  - Even when attacker can select messages m', receive $\sigma'=S_s(m')$
  - For any message except chosen m

Digital Signature Scheme Definition

Definition 1.4 (Signature scheme and its correctness). *A* signature scheme *is defined by a tuple of three efficient (PPT) algorithms, $\mathcal{S} = (\mathcal{KG}, \mathcal{S}ign, \mathcal{V}erify)$, and a set M of messages, such that:*

$\mathcal{KG}$ *is a randomized algorithm that maps a unary string (security parameter 1^l) to a pair of binary strings $(\mathcal{KG}.s(1^l), \mathcal{KG}.v(1^l))$.*

$\mathcal{S}ign$ *is an algorithm[8] that receives two binary strings as input, a signing key $s \in \{0,1\}^*$ and a message $m \in M$, and outputs another binary string $\sigma \in \{0,1\}^*$. We call σ the* signature *of m using signing key s.*

$\mathcal{V}erify$ *is a predicate that receives three binary strings as input: a verification key v, a message m, and σ, a purported signature over m. $\mathcal{V}erify$ should output* TRUE *if σ is the signature of m using s, where s is the signature key corresponding to v (generated with v).*

Usually, M is a set of binary strings of some length. If M is not defined, then this means that any binary string may be input, i.e., the same as $M = \{0,1\}^*$.

We say that a signature scheme $(\mathcal{KG}, \mathcal{S}ign, \mathcal{V}erify)$ is correct, *if for every security parameter 1^l holds:*

$$\left(\forall (s,v) \xleftarrow{\$} \mathcal{KG}(1^l),\ m \in M\right) \mathcal{V}erify_v(m, \mathcal{S}ign_s(m)) = \text{‘}Ok\text{’} \qquad (1.31)$$

Digital Signature Scheme Security

Algorithm 1 The existential unforgeability game $EUF^{Sign}_{\mathcal{A},\mathcal{S}}(1^l)(1^l)$ between signature scheme $\mathcal{S} = (\mathcal{KG}, \mathcal{Sign}, \mathcal{Verify})$ and adversary $\mathcal{A}$.

$(s, v) \xleftarrow{\$} \mathcal{S}.\mathcal{KG}(1^l)$;
$(m, \sigma) \xleftarrow{\$} \mathcal{A}^{\mathcal{S}.\mathcal{Sign}_s(\cdot)}(v, 1^l)$;
return $(\mathcal{S}.\mathcal{Verify}_v(m, \sigma) \wedge (\mathcal{A}$ didn't request $S_s(m)))$;

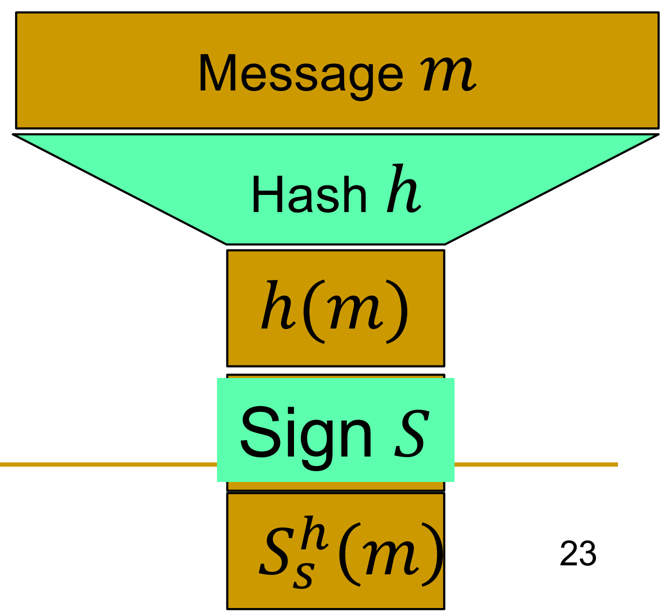
Definition 1.6. *The* existential unforgeability advantage function *of adversary $\mathcal{A}$ against signature scheme $\mathcal{S}$ is defined as:*

$$\varepsilon^{EUF-Sign}_{\mathcal{S},\mathcal{A}}(1^l) \equiv \Pr\left(EUF^{Sign}_{\mathcal{A},\mathcal{S}}(1^l)(1^l) = \textsc{True}\right) \tag{1.32}$$

Where the probability is taken over the random coin tosses of $\mathcal{A}$ and of $\mathcal{S}$ during the run of $EUF^{Sign}_{\mathcal{A},\mathcal{S}}(1^l)$ with input (security parameter) 1^l, and $EUF^{Sign}_{\mathcal{A},\mathcal{S}}(1^l)$ is the game defined in Algorithm 1.

RSA Signatures

- Secret signing key s, public verification key v
- Short (<n) messages: RSA signing with message recovery
- σ = RSA.$S_s(m) = m^s \bmod n$,
  RSA.$V_v(m, \sigma) = \{ OK \text{ if } m = \sigma^v \bmod n; \text{ else, } FAIL \}$
- Long messages: ??
  - Hint: use collision resistant hash function (CRHF)
  - σ = RSA.$S_s(m) = h(m)^s \bmod n$,
    RSA.$V_v(m, \sigma) = \{ OK \text{ if } h(m) = \sigma^v \bmod n; \text{ else, } FAIL \}$



Discrete-Log Digital Signature?

- RSA allowed encryption and signing… based on assuming factoring is hard
- Can we sign based on assuming discrete log is hard?
- Most well-known, popular scheme: DSA
  - Digital Signature Algorithm, by NSA/NIST
  - Details: crypto course

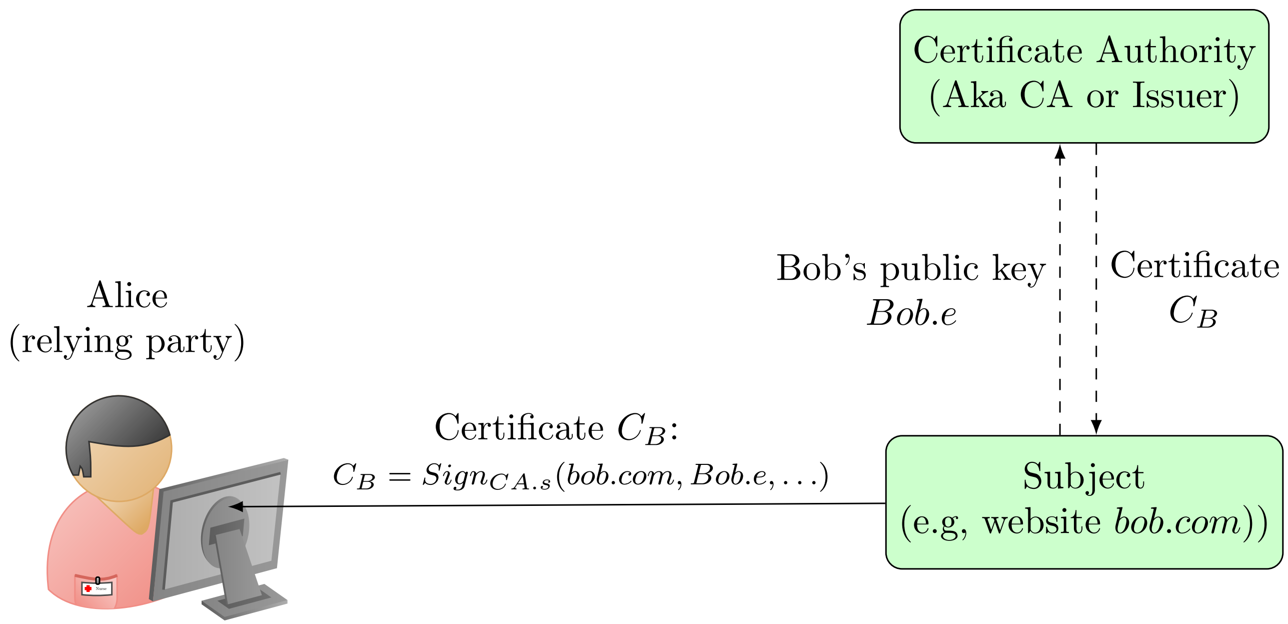
Public Key Infrastructure
PKI

Public keys are very useful…

- Secure web connections
- Software signing (against malware)
- Secure messaging, email
- Cryptocurrency and blockchains.
- But …
  - How do we know the PK of an entity?
    - Mainly: signed by a **trusted** **Certificate Authority**
    - E.g., in TLS, browsers maintain list of ‘root CAs’

Public Key Certificates & Authorities

- *Certificate:* signature by Issuer / Certificate Authority (CA) over subject's public key and attributes
- Attributes: identity (ID) and others…
  - Validated by CA (liability?)
  - Used by **relying party** for decisions (e.g., use this website?)



Certificates are all about **Trust**

- Certificate: $C_{Bob} = Sign_{CA.s}(Bob.com, Bob.e, \ldots)$
  - CA attests that Bob's public key is $Bob.e$
- Do we **trust** this attestation to be true?
- Special case of **trust management**
  - Important problem far beyond PKI… still not resolved !

Rogue Certificates

- Rogue cert: equivocating or misleading (domain) name
- Attacker goals:
  - Impersonate: web-site, phishing email, signed malware..
  - Equivocating (same name): circumvent name-based security mechanisms, such as *Same-Origin-Policy (SOP), blacklists, whitelists*, *access-control* …
  - Name may be misleading even if not equivocating
- Types of misleading names ('cybersquatting'):
  - Combo names: bank.com vs. accts-bank.com, bank.accts.com, …
  - Domain-name hacking: accts.bank.com vs. accts-bank.com, … or accts-bank.co
  - Homographic: paypal.com [l is L] vs. paypaI.com [i is I]
  - Typo-squatting: bank.com vs. banc.com, baank.com, banl.com,…

PKI Failures

- Although the signature over the certificate verifies correctly, there is still a failure and the certificate must be revoked.
  - This is called a PKI failure.
- PKI failures include:
  - Subject key exposure.
  - CA failure.
  - Cryptanalysis certificate forgery.
    - Find collisions in the hash function used in the HtS paradigm,
    - or exploit some vulnerability in the digital signature scheme used for signing.

Some Infamous PKI Failures

2001	VeriSign: attacker gets code-signing certs
2008	Thawte: email-validation (attackers' mailbox)
2008,11	Comodo not performing domain validation
2011	DigiNotar compromised, 531 rogue certs (discovered); a rogue cert for **.google.com* used for MitM against 300,000 Iranian users.
2011	TurkTrust issued intermediate-CA certs to users
2012	Trustwave issued intermediate-CA certificate for eavesdropping
2013	ANSSI, the French Network and Information Security Agency, issued intermediate-CA certificate to MitM traffic management device
2014	India CCA / NIC compromised (and issued rogue certs)
2015	CNNIC (China) issued CA-cert to MCS (Egypt), who issued rogue certs. Google and Mozilla removed CNNIC from their root programs.
2013-17	Audio driver of Savitech install root CA in Windows
2015,17	Symantec issued unauthorized certs for over 176 domains, causing removal from all root programs.
2019	Mozilla, Google *browsers* block *customer-installed* Kazakhstan root CA (Qaznet)
2019	Mozilla, Google revoke intermediate-CA of DarkMatter, and refuse to add them to root program

PKI Goals/Requirements

Trustworthy issuers: Trust anchor/root CAs and Intermediary CAs; Limitations on Intermediary CAs (e.g., restricted domain names)

Accountability: identify issuer of given certificate

Timeliness: limited validity period, timely **revocation**

Transparency: public log of all certificate; no ‘hidden’ certs!

Non-Equivocation: one entity – one certificate

Privacy: why should CA know which site I use?

Covered Material From the Textbook

- Chapter 1: Section: 1.4
- Chapter 6: Sections 6.4, 6.5 (except 6.5.6 and 6.5.7), and 6.6 (except RSA with message recovery)
- Chapter 8: Section 8.1

Thank You!