CSE 3400 - Introduction to Computer & Network Security
(aka: Introduction to Cybersecurity)

# Lecture 11
# Public Key Cryptography– Part II

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From Textbook Slides by Prof. Amir Herzberg
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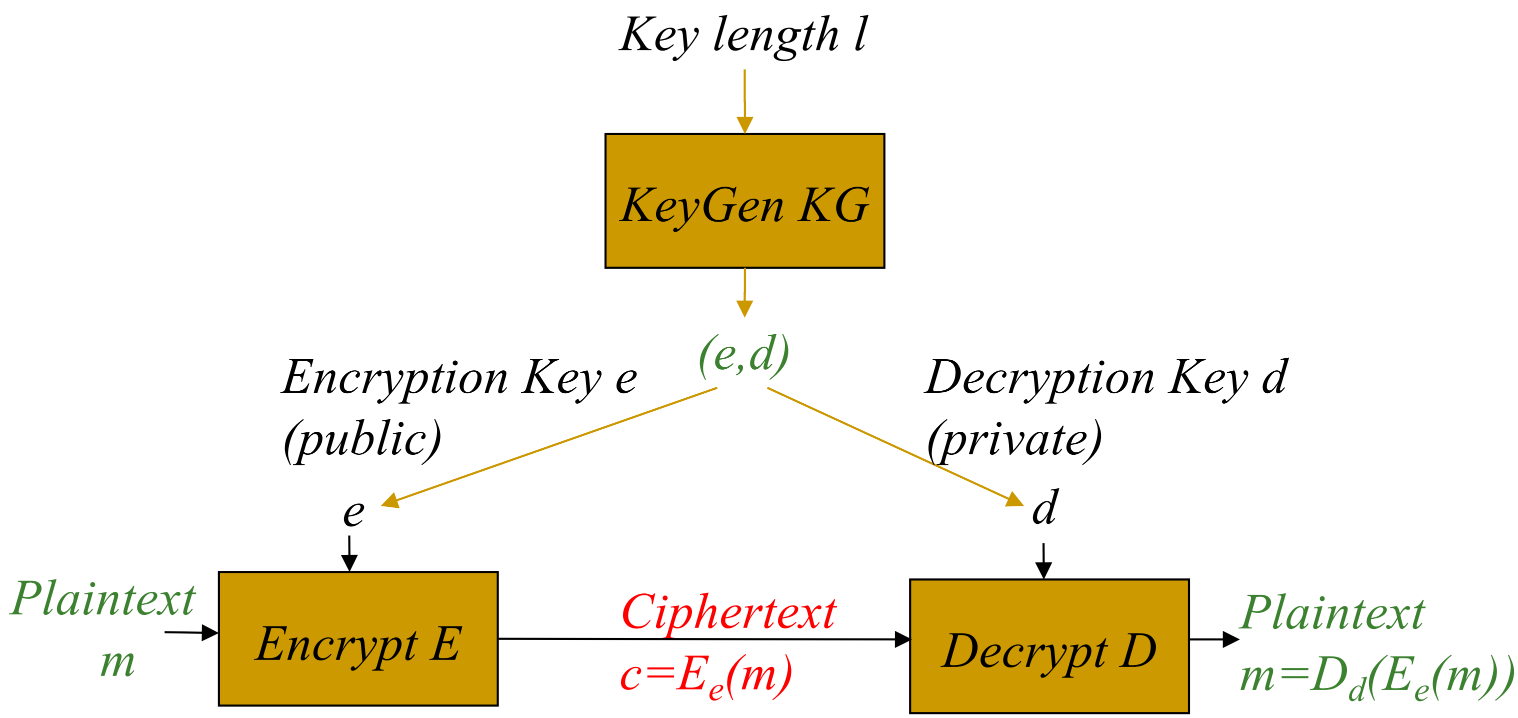
# Outline

- Public key encryption.
- Digital signatures.

# Public Key Encryption

# Public Key Encryption



# Public Key Encryption IND-CPA Security

$T^{IND-CPA}_{\mathcal{A},\langle KG,E,D\rangle}(b,n)$ {
  $(e,d) \stackrel{\$}{\leftarrow} KG(1^n)$
  $(m_0,m_1) \leftarrow \mathcal{A}(\text{'Choose'}, e)$ s.t. $|m_0| = |m_1|$
  $c^* \leftarrow E_e(m_b)$
  $b^* = \mathcal{A}(\text{'Guess'}, (c^*, e))$
  Return $b^*$
}

**Definition 2.10** (PKC IND-CPA)**.** *Let $\langle KG, E, D\rangle$ be a public-key cryptosystem. We say that $\langle KG, E, D\rangle$ is* IND-CPA*, if every efficient adversary $\mathcal{A} \in PPT$ has negligible advantage $\varepsilon^{IND-CPA}_{<KG,E,D>,\mathcal{A}}(n) \in NEGL(n)$, where:*

$$\varepsilon^{IND-CPA}_{\langle KG,E,D\rangle,\mathcal{A}}(n) \equiv \Pr\left[T^{IND-CPA}_{\mathcal{A},\langle KG,E,D\rangle}(1,n) = 1\right] - \Pr\left[T^{IND-CPA}_{\mathcal{A},\langle KG,E,D\rangle}(0,n) = 1\right] \tag{2.35}$$

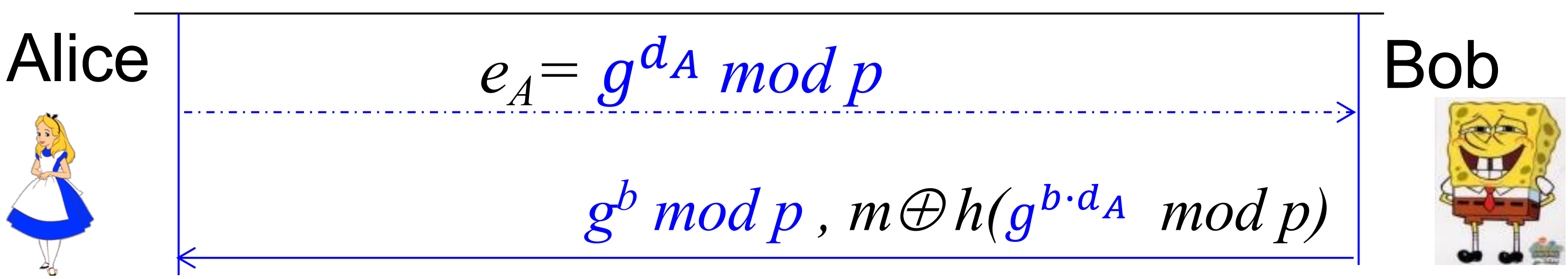
*Where the probability is over the random coin tosses in IND-CPA (including of $\mathcal{A}$ and $E$).*

# Discrete Log-based Encryption

- We will explore two flavors:
  - An adaptation of DH key exchange protocol to perform encryption.
  - ElGamal encryption scheme.

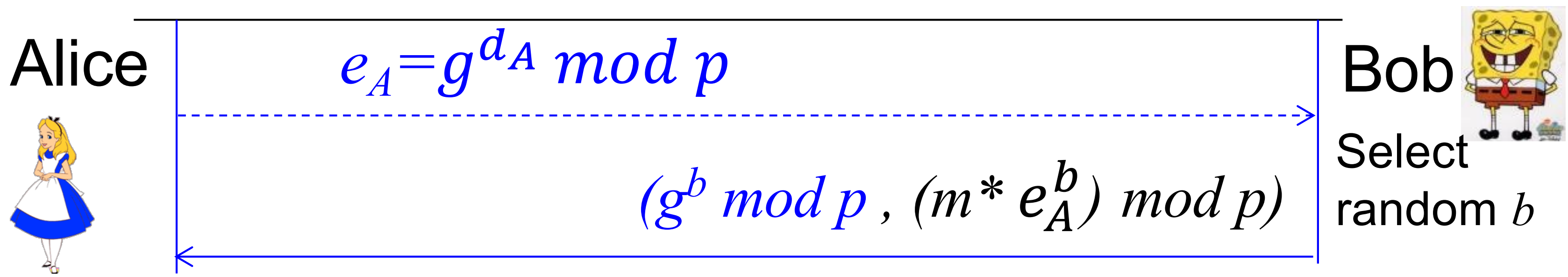
# Turning [DH] to Public Key Cryptosystem

- Solves dependency on DDH assumption; secure under the (weaker) CDH assumption.
- To encrypt message m to Alice:
  - Bob selects random $b$
  - Sends: $g^b \ mod\ p\ ,\ m \oplus h((e_A)^b) = m \oplus h(g^{b \cdot d_A}\ mod\ p)$
  - Secure if $h(g^{b \cdot d_A}\ mod\ p)$ is pseudo-random



# ElGamal Public Key Encyption

- Variant of [DH] PKC: Encrypt by multiplication, not XOR
- To encrypt message $m$ to Alice, whose public key is $e_A = g^{d_A} \bmod p$:
  - Bob selects random $b$
  - Sends: $g^b \bmod p$ , $m^*(e_A)^b = m^* g^{b \cdot d_A} \bmod p$



# ElGamal Public Key Encryption

- Encryption:

$$E^{EG}_{e_A}(m) \leftarrow \left\{ \left(g^b \mod p\ ,\ m \cdot e_A^b \mod p\right) | b \stackrel{\$}{\leftarrow} [2, p-1] \right\}$$

- Decryption:

$$D_{d_A}(x, y) = x^{-d_A} \cdot y \mod p$$

- Correctness:

$$\begin{aligned} D_{d_A}(g^b \mod p\ ,\ m \cdot e_A^b \mod p) &= \\ &= \left[\left(g^b \mod p\right)^{-d_A} \cdot \left(m \cdot \left(g^{d_A}\right)^b \mod p\right)\right] \mod p \\ &= \left[g^{-b \cdot d_A} \cdot m \cdot g^{b \cdot d_A}\right] \mod p \\ &= m \end{aligned}$$

# ElGamal Public Key Cryptosystem

- Problem: $g^{b \cdot d_A}$ $mod\ p$ may leak bit(s)…
- `Classical' DH solution: securely derive a key: $h\left(g^{a_i b_i} mod\ p\right)$
- El-Gamal's solution: use a group where DDH believed to hold
  - Note: message must be encoded as member of the group!
  - So why use it? Some special properties…

# ElGamal PKC: homomorphism

- Multiplying two ciphertexts produces a ciphertext of the multiplication of the two plaintexts.
- Given two ciphertexts:
  - $E_{e_A}(m_1) = (x_1, y_1) = (g^{b_1} \bmod p, m_1 * g^{b_1 \cdot d_A} \bmod p)$
  - $E_{e_A}(m_2) = (x_2, y_2) = (g^{b_2} \bmod p, m_2 * g^{b_2 \cdot d_A} \bmod p)$
- $Mult\big((x_1, y_1), (x_2, y_2)\big) \equiv (x_1 x_2, y_1 y_2)$
- Homomorphism:
- $= (g^{b_1+b_2} \bmod p, m_1 \cdot m_2 * g^{(b_1+b_2) \cdot d_A} \bmod p) =$
  $= E_{e_A}(m_1 \cdot m_2)$
- ➔ compute $E_{e_A}(m_1 \cdot m_2)$ from $E_{e_A}(m_1)$, $E_{e_A}(m_1)$

# RSA Public Key Encryption

2002 Turing Award

- First proposed – and still widely used
- Not really covered in this course – take crypto!
- Select two large primes $p,q$ ; let $n=pq$
- Select prime $e$ (public key: $<n,e>$)
  - Or co-prime with $\Phi(n) = (p-1)(q-1)$
- Let private key be $d=e^{-1} \bmod \Phi(n)$ (i.e., $ed=1 \bmod \Phi(n)$)
- Encryption: $RSA.E_{e,n}(m)=m^e \bmod n$
- Decryption: $RSA.D_{d,n}(c)=c^d \bmod n$
- Correctness: $D_{d,n}(E_{e,n}(m)) = (m^e)^d = m^{ed} = m \bmod n$
  - Intuitively: $ed=1 \bmod \Phi(n) \rightarrow m^{ed} = m \bmod n$
- But why? Remember Euler's theorem.

# RSA Public Key Cryptosystem

- Correctness: $D_{d,n}(E_{e,n}(m)) = m^{ed} \; mod \; n$
- $m^{ed} = m^{ed} = m^{1+l\,\Phi(n)} = m\, m^{l\,\Phi(n)} = m\,(m^{\Phi(n)})^{l}$
- $m^{ed} \; mod \; n = m\,(m^{\Phi(n)} \; mod \; n)^{l} \; mod \; n$
- Eulers'Theorem: $m^{\Phi(n)} \; mod \; n = 1 \; mod \; n$
- ➔ $D_{d,n}(E_{e,n}(m)) = m^{ed} \; mod \; n = m \; 1^{l} \; mod \; n = m$
- Comments:
  - $m < n$ ➔ $m = m \; mod \; n$
  - Eulers' Theorem holds (only) if $m$, $n$ are co-primes
  - If not co-primes? Use Chinese Reminder Theorem
    - A nice, not very complex argument
    - But: beyond our scope – take Crypto!

# The RSA Problem and Assumption

- RSA problem: Find $m$ , given $(n,e)$ and 'ciphertext' value $c=m^e \bmod n$
- RSA assumption: if $(n,e)$ are chosen `correctly', then the RSA problem is `hard'
  - I.e., no efficient algorithm can find $m$ with non-negligible probability
  - For `large' $n$ and $m \overset{\$}{\leftarrow} \{1, \ldots, n\}$
- RSA and factoring
  - Factoring alg ➔ alg to 'break' RSA
  - Algorithm to find RSA private key ➔ factoring alg
  - But: RSA-breaking may *not* allow factoring

# RSA PKC Security

- It is a deterministic encryption scheme → cannot IND-CPA secure.
- RSA assumption does not rule out exposure of partial information about the plaintext.
- It is not CCA secure.

*A solution: apply a random padding to the plaintext then encryption using RSA.*

# Padding RSA

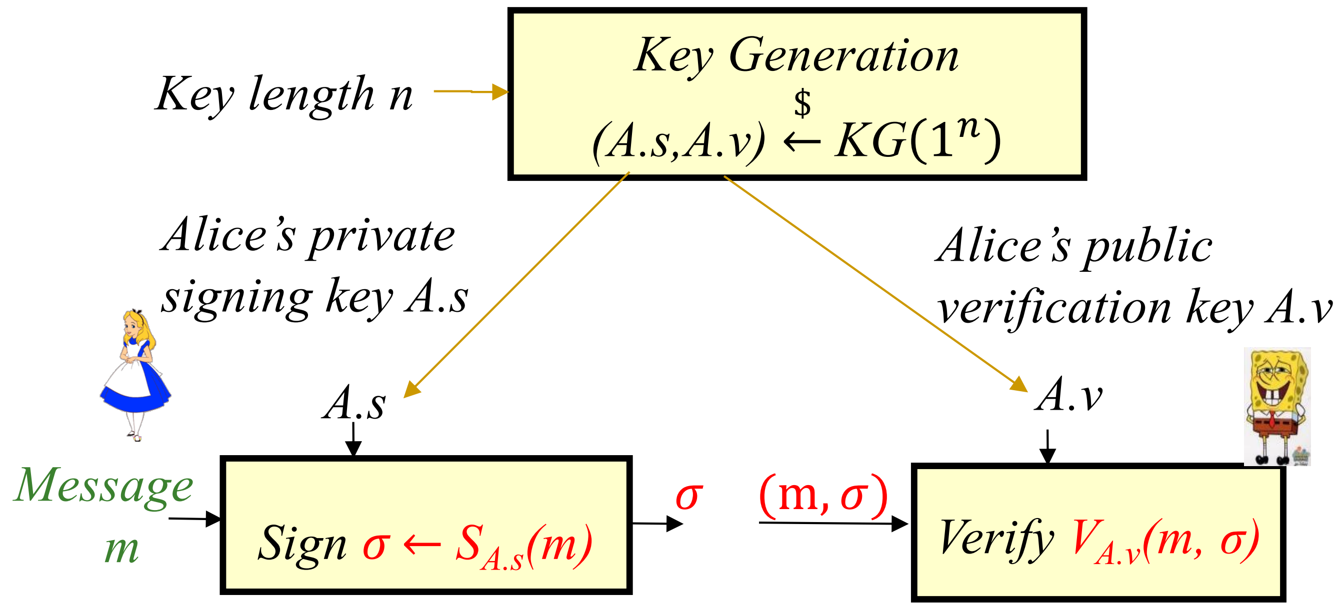
- Pad and Unpad functions: $m = Unpad(Pad(m;r))$
  - Encryption with padding: $c = [Pad(m,r)]^e \bmod n,$
  - Decryption with unpad: $m = Unpad(c^d \bmod n)$
- Required to…
  - Add randomization
    - Prevent detection of repeating plaintext
  - Prevent ‘related message’ attack (to allow use of tiny $e$)
  - Detect, prevent (some) chosen-ciphertext attacks
- Early paddings schemes subject to CCA attacks
  - Even ‘Feedback-only CCA’ (aware of unpad failure)

# How does Bob know Alice's public key?

- Depends on threat model…
  - Passive (`eavesdropping`) adversary: just send it
  - Man-in-the-Middle (MITM): **authenticate**
- Authenticate – how?
  - MAC: requires shared secret key
  - **Public key signature scheme**: authenticate using public key
  - Certificate: public key of entity – **signed** by **certificate authority (CA)**

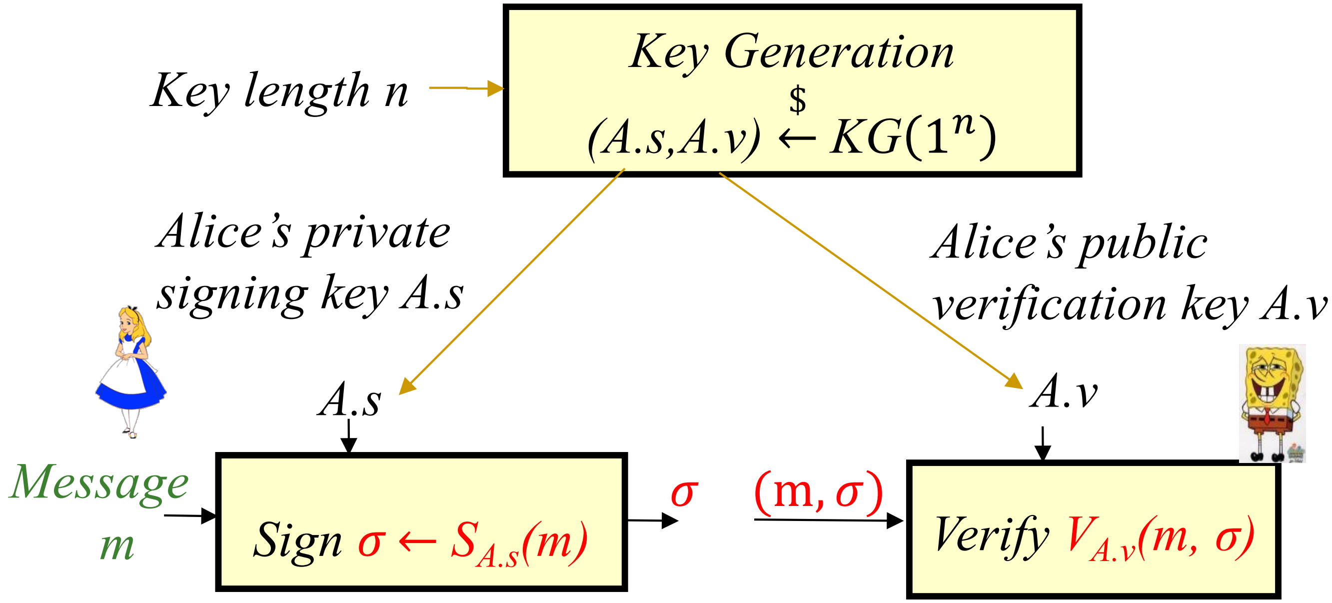
# Digital Signature

# Public Key Digital Signatures




- Sign using a private, secret signature key ($A.s$ for Alice)
- Validate using a <u>public</u> key ($A.v$ for Alice)
- Everybody can validate signatures at any time
  - Provides authentication, integrity **<u>and</u>** evidence / non-repudiation
  - MAC: 'just' authentication+integrity, no evidence, can repudiate

# Digital Signatures Security: Unforgeability




- Unforgeability: given $v$, attacker should be unable to find **any** 'valid' $(m, \sigma)$, i.e., $V_v(m, \sigma)=OK$
  - Even when attacker can select messages $m'$, receive $\sigma'=S_s(m')$
  - For any message except chosen $m$

# Digital Signature Scheme Definition

**Definition 1.4** (Signature scheme and its correctness)**.** *A* signature scheme *is defined by a tuple of three efficient (PPT) algorithms,* $\mathcal{S} = (\mathcal{KG}, \mathcal{S}ign, \mathcal{V}erify)$, *and a set* $M$ *of messages, such that:*

$\mathcal{KG}$ *is a randomized algorithm that maps a unary string (security parameter* $1^l$*) to a pair of binary strings* $(\mathcal{KG}.s(1^l), \mathcal{KG}.v(1^l))$.

$\mathcal{S}ign$ *is an algorithm*[8] *that receives two binary strings as input, a signing key* $s \in \{0,1\}^*$ *and a message* $m \in M$*, and outputs another binary string* $\sigma \in \{0,1\}^*$. *We call* $\sigma$ *the* signature *of* $m$ *using signing key* $s$.

$\mathcal{V}erify$ *is a predicate that receives three binary strings as input: a verification key* $v$*, a message* $m$*, and* $\sigma$*, a purported signature over* $m$*.* $\mathcal{V}erify$ *should output* True *if* $\sigma$ *is the signature of* $m$ *using* $s$*, where* $s$ *is the signature key corresponding to* $v$ *(generated with* $v$*).*

*Usually,* $M$ *is a set of binary strings of some length. If* $M$ *is not defined, then this means that any binary string may be input, i.e., the same as* $M = \{0,1\}^*$.

*We say that a signature scheme* $(\mathcal{KG}, \mathcal{S}ign, \mathcal{V}erify)$ *is* correct*, if for every security parameter* $1^l$ *holds:*

$$\left(\forall (s,v) \xleftarrow{\$} \mathcal{KG}(1^l),\ m \in M\right) \mathcal{V}erify_v(m, \mathcal{S}ign_s(m)) = \text{‘Ok’} \qquad (1.31)$$

# Digital Signature Scheme Security

**Algorithm 1** The existential unforgeability game $EUF^{Sign}_{\mathcal{A},\mathcal{S}}(1^l)(1^l)$ between signature scheme $\mathcal{S} = (\mathcal{KG}, \mathcal{Sign}, \mathcal{Verify})$ and adversary $\mathcal{A}$.

$(s, v) \xleftarrow{\$} \mathcal{S}.\mathcal{KG}(1^l)$ ;
$(m, \sigma) \xleftarrow{\$} \mathcal{A}^{\mathcal{S}.\mathcal{Sign}_s(\cdot)}(v, 1^l)$;
**return** $(\mathcal{S}.\mathcal{Verify}_v(m, \sigma) \wedge (\mathcal{A}$ didn't request $S_s(m)))$;

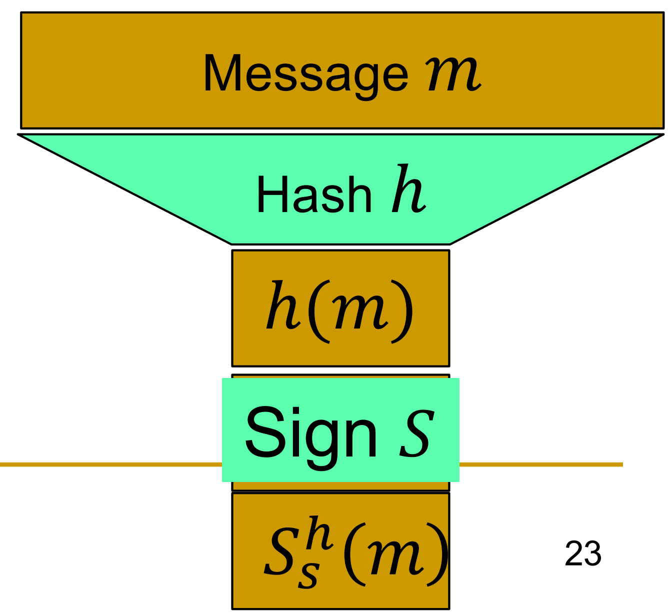
**Definition 1.6.** *The* existential unforgeability advantage function *of adversary $\mathcal{A}$ against signature scheme $\mathcal{S}$ is defined as:*

$$\varepsilon^{EUF-Sign}_{\mathcal{S},\mathcal{A}}(1^l) \equiv \Pr\left(EUF^{Sign}_{\mathcal{A},\mathcal{S}}(1^l)(1^l) = \text{TRUE}\right) \tag{1.32}$$

*Where the probability is taken over the random coin tosses of $\mathcal{A}$ and of $\mathcal{S}$ during the run of $EUF^{Sign}_{\mathcal{A},\mathcal{S}}(1^l)$ with input (security parameter) $1^l$, and $EUF^{Sign}_{\mathcal{A},\mathcal{S}}(1^l)$ is the game defined in Algorithm 1.*

# RSA Signatures

- Secret signing key $s$, public verification key $v$
- Short (<n) messages: RSA signing with message recovery
- $\sigma$ = RSA.$S_s(m) = m^s \mod n$,
  RSA.$V_v(m, \sigma) = \{ OK \text{ if } m = \sigma^v \mod n; \text{ else, } FAIL \}$
- Long messages: ??
  - Hint: use collision resistant hash function (CRHF)
  - $\sigma$ = RSA.$S_s(m) = h(m)^s \mod n$,
    RSA.$V_v(m, \sigma) = \{ OK \text{ if } h(m) = \sigma^v \mod n; \text{ else, } FAIL \}$



# Discrete-Log Digital Signature?

- RSA allowed encryption and signing… based on assuming factoring is hard
- Can we sign based on assuming discrete log is hard?
- Most well-known, popular scheme: DSA
  - Digital Signature Algorithm, by NSA/NIST
  - Details: crypto course

# Covered Material From the Textbook

- Chapter 1, Section: 1.2.3
- Chapter 2, Sections 2.7.3
- Chapter 6, Sections 6.4, 6.5 (except 6.5.6 and 6.5.7), and 6.6 (except RSA with message recovery)

# Thank You!