CSE 3400 - Introduction to Computer & Network Security
(aka: Introduction to Cybersecurity)

Lecture 6

Hash Functions – Part I

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From Textbook Slides by Prof. Amir Herzberg
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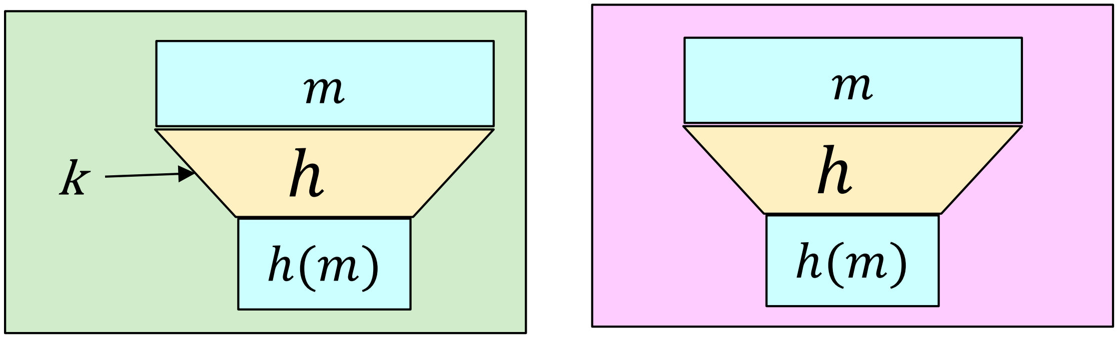
Outline

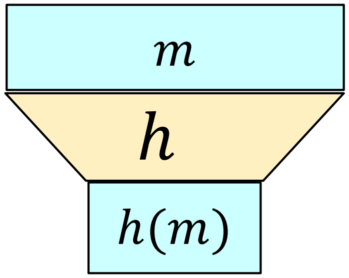
- Introduction and motivation.
- Collision resistant hash functions (CRHF).
- CRHF applications.
- Weaker notions of security.
  - TCR, SPR, OWF.
- Randomness extraction.
- The random oracle model.

Hash Functions

- Input m: binary strings
- Output $h(m)$:
  - 'Short' (n-bit) binary strings
    - Aka **message digest**
- Efficiently computable
- Applications: cryptography, security, efficiency
- Keyed $h_k(m)$, where the key is public, or unkeyed $h(m)$



Hash functions: simple examples



- For simplicity: input m is decimal integer
  - View as string of (three) digits
  - For example, $m = 127 \rightarrow m_1 = 1, m_2 = 2, m_3 = 7$
- Least Significant Digit hash:

$$h_{LSD}(m) = m_3$$

- Sum hash: $h_{Sum}(m) = (m_1 + m_2 + m_3) \; mod \; 10$
- Exercise:

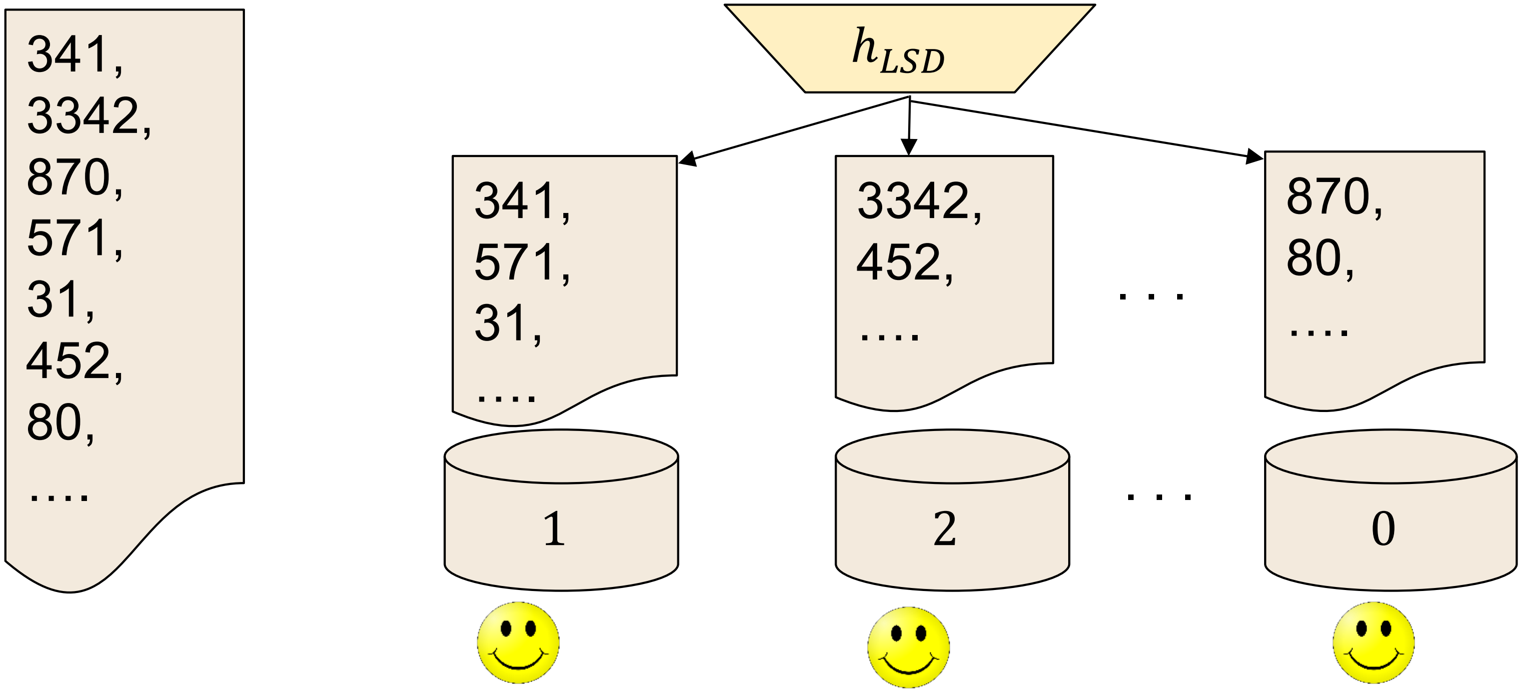
$h_{LSD}(117) = \underline{7}$

$h_{Sum}(117) = \underline{9}$

Note: the above are insecure hash functions, these are just toy examples to grasp the concept of hashing.

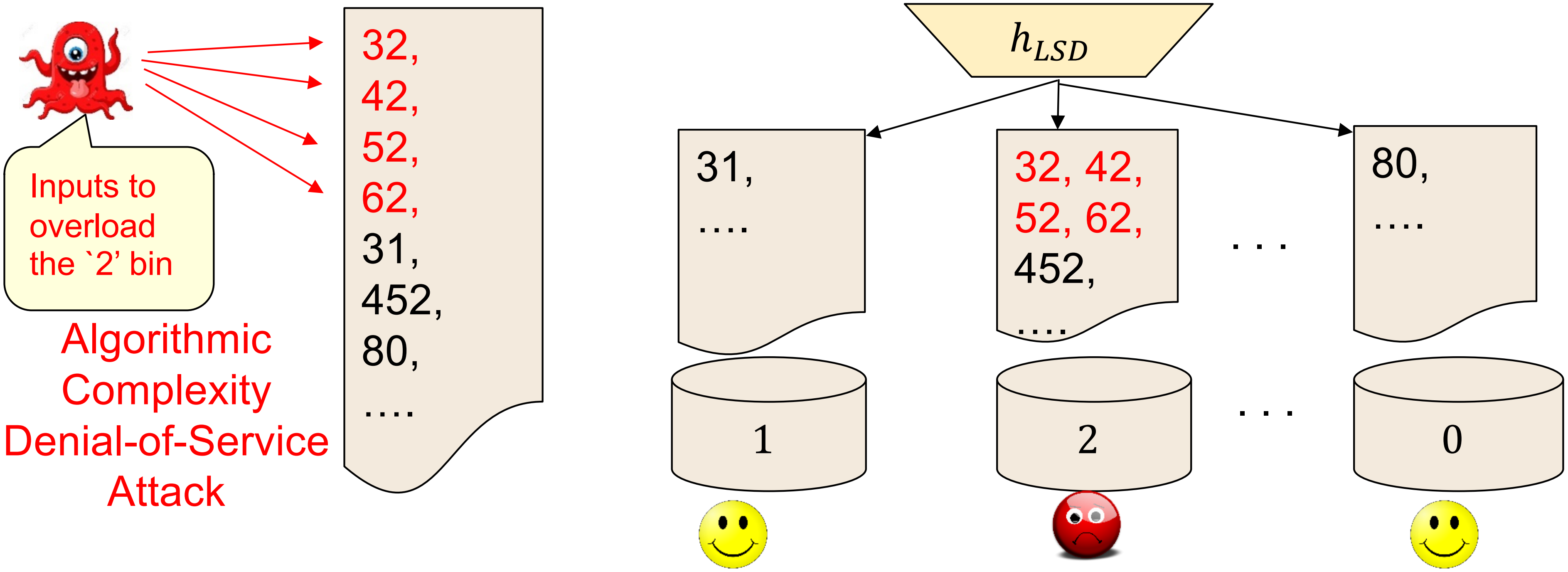
Motivation: Hashing for efficiency

- Input: large set (e.g., integers or strings)
- Goal: map `randomly’ to few bins
  - E.g., to ensure efficiency – load balancing, etc.



Collisions?

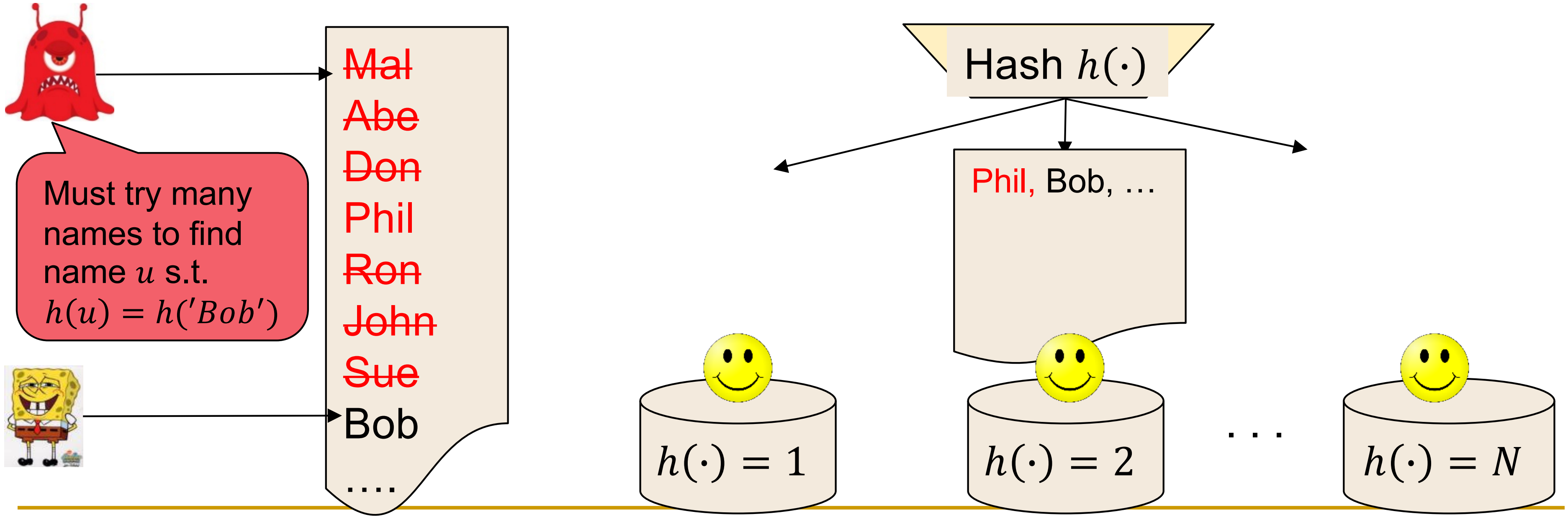
- Input: large set (e.g., integers or strings)
- Goal: map `randomly’ to few bins
  - E.g., to ensure efficiency – load balancing, etc.
  - Adversary chooses inputs that hash to same bin



Algorithmic Complexity Denial-of-Service Attack

Security Goal: Collision Resistance

- A **collision:** two inputs (names) with same hash: $h('Bob')=h('Phil')$
- Every hash has collisions, since |input|>>|output| !
- Collision resistance: hard to **find** collisions
  - Note: attacker can always try names randomly until a collision is found
  - But this should be ineffective: must try about (on average) N names (number of bins)



Must try many names to find name u s.t. $h(u) = h('Bob')$

~~Mal~~
~~Abe~~
~~Don~~
Phil
~~Ron~~
~~John~~
~~Sue~~
Bob
….

Hash $h(\cdot)$

Phil, Bob, …

$h(\cdot) = 1$

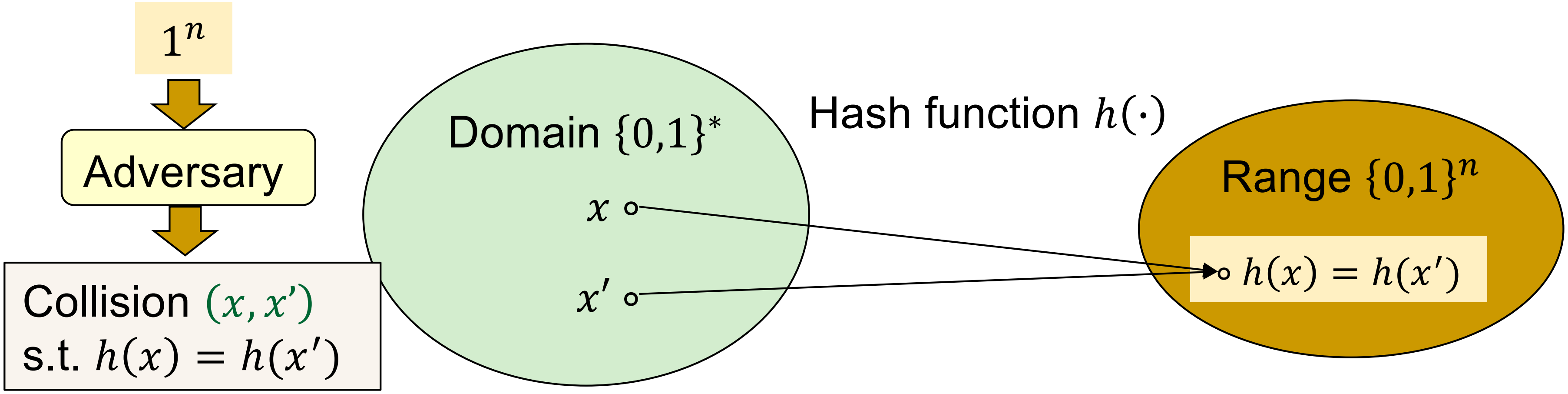
$h(\cdot) = 2$

. . .

$h(\cdot) = N$

Collision Resistant Hash Function (CRHF)

- h is CRHF if it is hard to **find** collisions $h(x)=h(x')$
  - Note: attacker can always try inputs randomly till finding collisions
  - But this should be ineffective: must try about $|Range|$ values
- Hard means that the probability that the attacker succeeds in finding a collision is negligible.



Collision Resistant Hash Function (CRHF)

- h is CRHF if it is hard to **find** collisions $h(x)$=$h(x')$
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Definition 4.1 (Keyless Collision Resistant Hash Function (CRHF)). *A keyless hash function $h^{(n)}(\cdot) : \{0,1\}^* \rightarrow \{0,1\}^n$ is* collision-resistant *if for every efficient (PPT) algorithm $\mathcal{A}$, the* advantage $\varepsilon^{CRHF}_{h,\mathcal{A}}(n)$ *is* negligible *in n, i.e., smaller than any positive polynomial for sufficiently large n (as $n \rightarrow \infty$), where:*

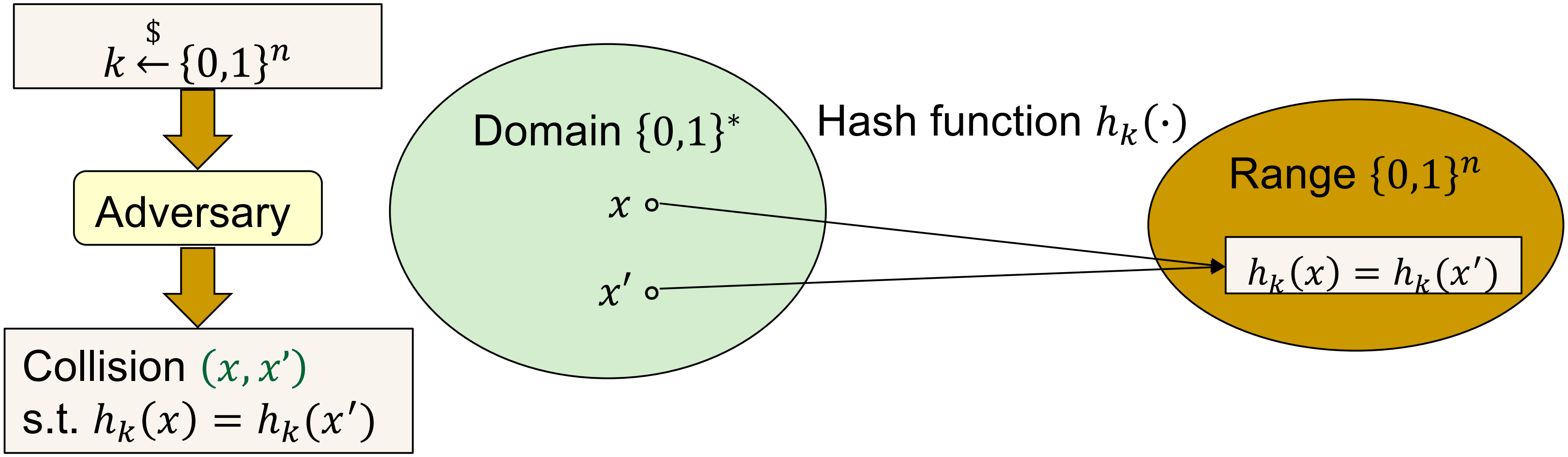
$$\varepsilon^{CRHF}_{h,\mathcal{A}}(n) \equiv \Pr\left[(x,x') \leftarrow \mathcal{A}(1^n) \text{ s.t. } (x \neq x') \wedge (h^{(n)}(x) = h^{(n)}(x')\right] \quad (4.1)$$

Where the probability is taken over the random coin tosses of $\mathcal{A}$.

Keyless CRHF **Do Not** Exist!

- |Range|<<|Domain| so there is a collision where $h(x')=h(x),\ x \neq x'$
- For a keyless CRHF is a PPT algorithm A that can always output a collision: $A(1^n) = \{return\ x, x'\}$
  - Proof: in textbook.
    - Intuitively, since the function is fixed (same input-output mapping), a collision instance can be hardcoded in the attacker algorithm and just out that collision and win the security game.
- Solutions:
  - keyed CRHF,
  - Use functions that support weak-collision-resistance,
  - or ignore! (more like asking if the collision is useful for the attacker?)

Keyed CRHF



$k \overset{\$}{\leftarrow} \{0,1\}^n$

Adversary

Collision (x, x')
s.t. $h_k(x) = h_k(x')$

Domain $\{0,1\}^*$

x

x'

Hash function $h_k(\cdot)$

Range $\{0,1\}^n$

$h_k(x) = h_k(x')$

Adversary knows k but **not in advance** – cannot `know` a collision

Often referred to as **ACR**-hash (**ANY**-collision resistance)

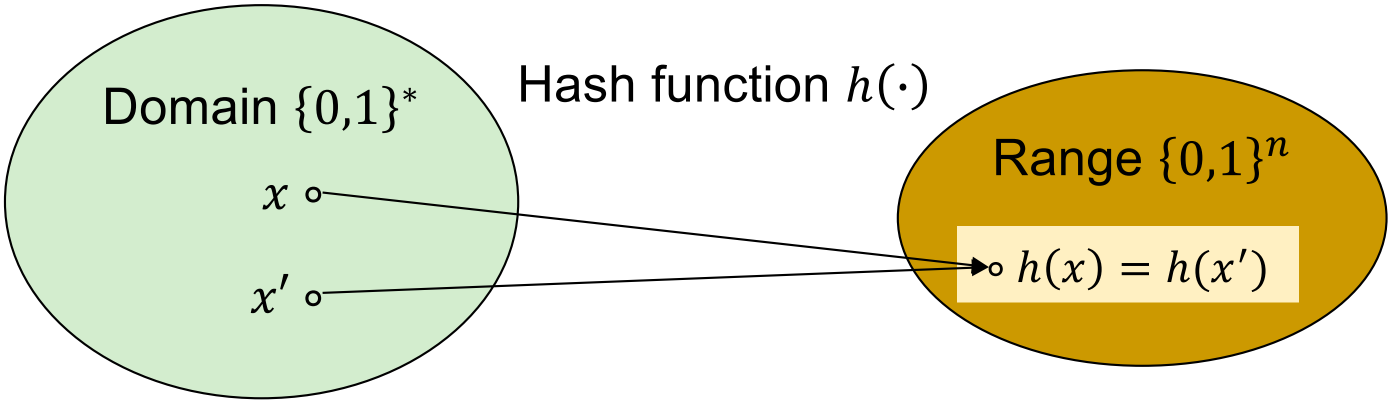
Keyed CRHF - Definition

Definition 4.3 (Keyed Collision Resistant Hash Function (CRHF))**.** *A keyed hash function* $h_k(\cdot) : \{0,1\}^* \times \{0,1\}^* \to \{0,1\}^*$ *is* collision-resistant *if for every efficient (PPT) algorithm* $\mathcal{A}$*, the* advantage $\varepsilon^{CRHF}_{h,\mathcal{A}}(n)$ *is* negligible *in* n*, i.e., smaller than any positive polynomial for sufficiently large* n *(as* $n \to \infty$*), where:*

$$\varepsilon^{CRHF}_{h,\mathcal{A}}(n) \equiv \Pr_{k \leftarrow \{0,1\}^n} \left[(x, x') \leftarrow \mathcal{A}(k) \;\; s.t. \;\; (x \neq x') \wedge (h_k(x) = h_k(x')\right] \quad (4.2)$$

Where the probability is taken over the random coin tosses of $\mathcal{A}$ *and the random choice of* k*.*

Generic Collision Attacks



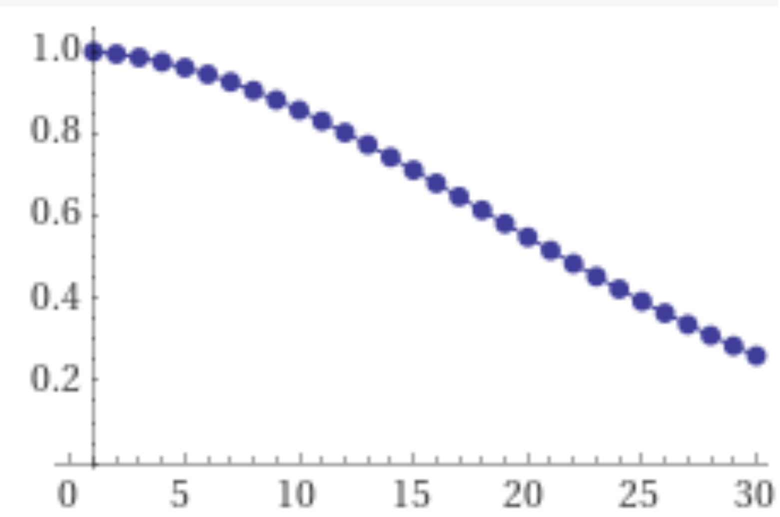
- An attacker that runs in exponential time can always find a collision (i.e., non PPT attacker)
  - Easy: find collisions in 2^n time by trying $2^n + 1$ distinct inputs (compute their hash and locate a collision).
- An attacker finds a collision with 2^{-n} probability (negligible probability).
  - Choose *x* and *x’* at random and check if they produce a collision.

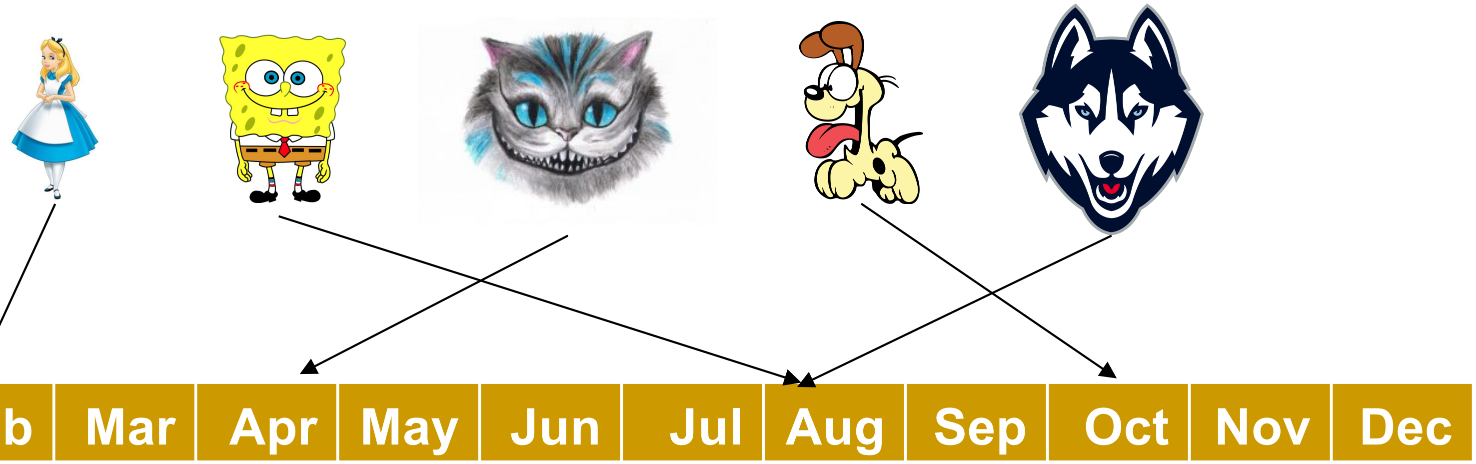
The Birthday Paradox

- **The birthday paradox** states that expected number q of hashes until a collision is found is $O(2^n)$ not $O(2^n)$.
  - It is $q \lessapprox 2^{n/2} \cdot \sqrt{\frac{\pi}{2}} \lessapprox 1.254 \cdot 2^{n/2}$
- For 80 bit of effective security, use n=160 !
  - So to defend against an attacker who can perform 2^{80} hashes set the digest length to be at least 160 bits.
    - So the range has a size of 2^{160} digests.
- Why? Intuition?

The Birthday Attack (‘Paradox’)

- Probability of NO birthday-collision:
    - Two persons: (364/365)
    - Three persons: (364/365)*(363/365)
    - …
    - n persons: $\prod_{i=1}^{n-1} \frac{365-i}{365}$





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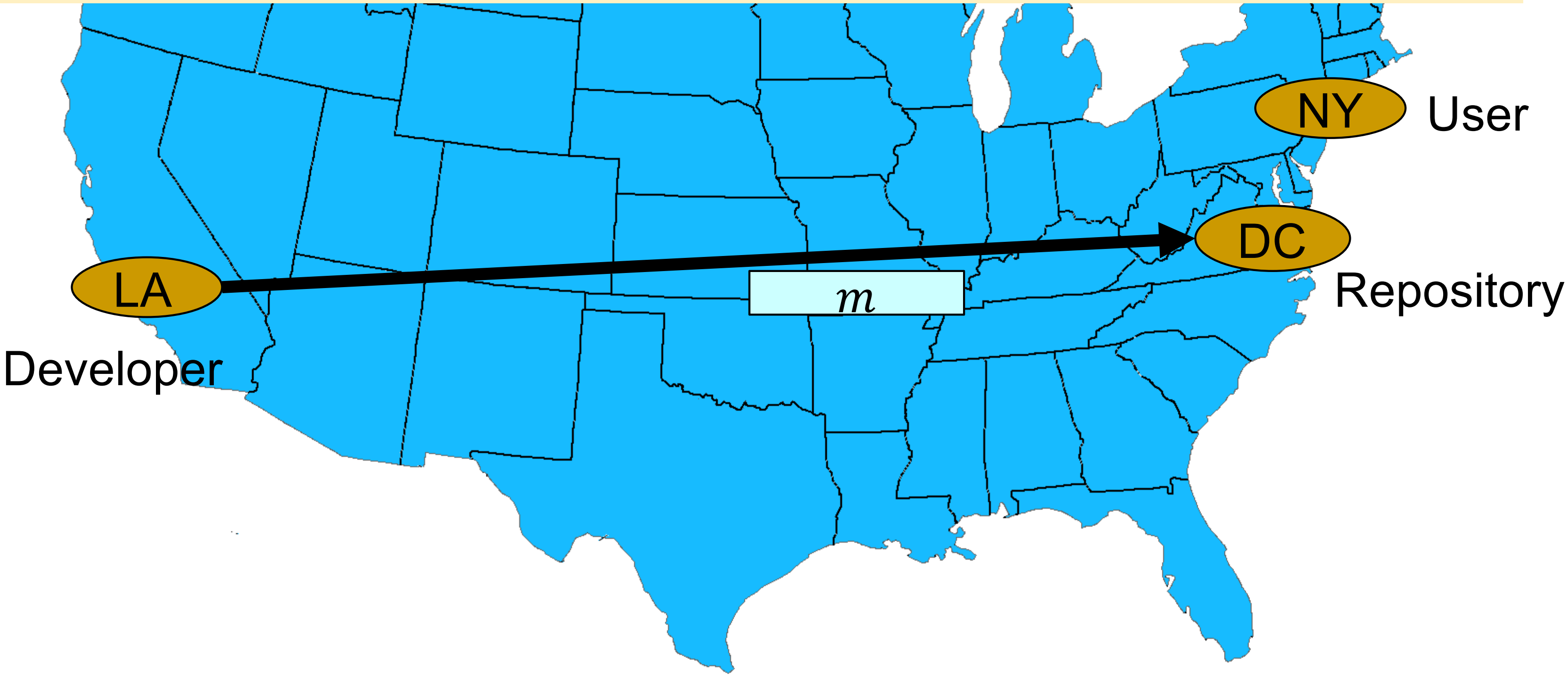
Collision-Resistance: Applications

- Integrity (of object / file / message)
  - Send *hash(m)* securely to validate m
  - Later we will see how a hash function can be used to construct a MAC (called HMAC).
- Hash-then-Sign
  - Instead of signing m sign *hash(m)*
    - More efficient!
    - We will explore this in detail once we study digital signatures.
- Blockchains
  - Later

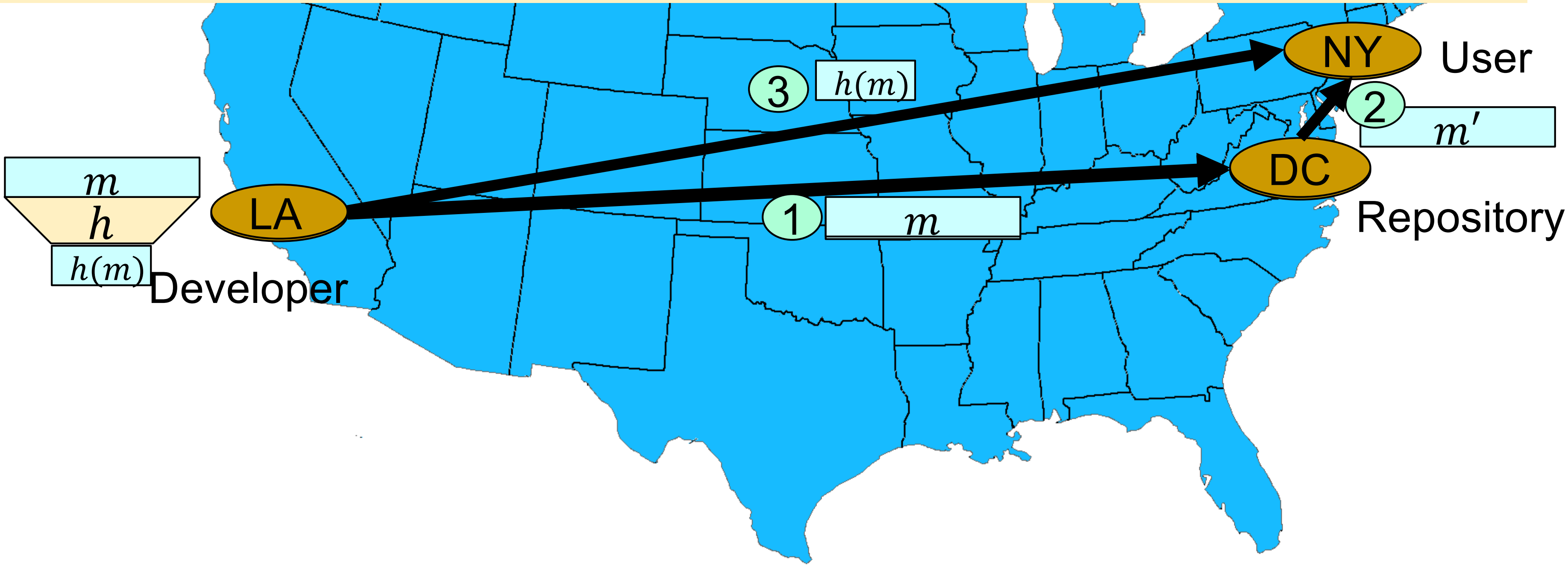
CRHF and Software Distribution

- Developer in LA develops large software m
- Repository in DC obtains copy of m
- User in NY wants to obtain m – securely and efficiently
  - Don't send m from LA to **both** NY and DC
- How?



CRHF: secure, efficient SW distribution

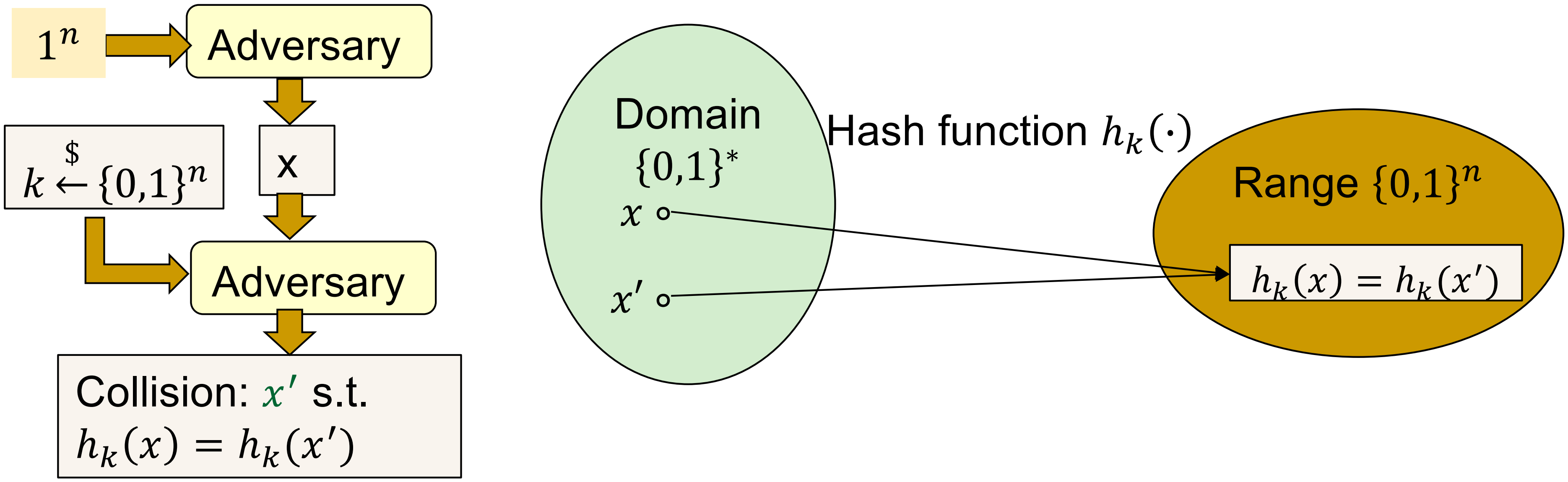
1. Repository in DC downloads software m from developer in LA
2. User download from (nearby) repository; receives m'
   - Is $m' = m$? User should validate! How?
3. User securely downloads $h(m)$ directly from developer
   - Digest $h(m)$ is short – much less overhead than downloading m
4. User validates: $h(m) = h(m')$➔$m = m'$



Weaker Notions of Security

- Collision resistance provide the strongest guarantee.
  - Gives more freedom to the adversary; the adversary wins if it finds any two inputs with the same digest.
    - No conditions on these two inputs other than being in the domain of the hash function.
- Weaker security notions (but sufficient for many applications):
  - Target collision resistance (TCR).
  - Second preimage resistance.
  - First preimage resistance.
- Birthday paradox (or attack) does not work against these weaker notions.
  - It is for collision resistance; find ***any*** two inputs that collide!

Target CRHF (TCR Hash Function)



1^n

Adversary

x

$k \overset{\$}{\leftarrow} \{0,1\}^n$

Adversary

Collision: x' s.t. $h_k(x) = h_k(x')$

Domain $\{0,1\}^*$

x

x'

Hash function $h_k(\cdot)$

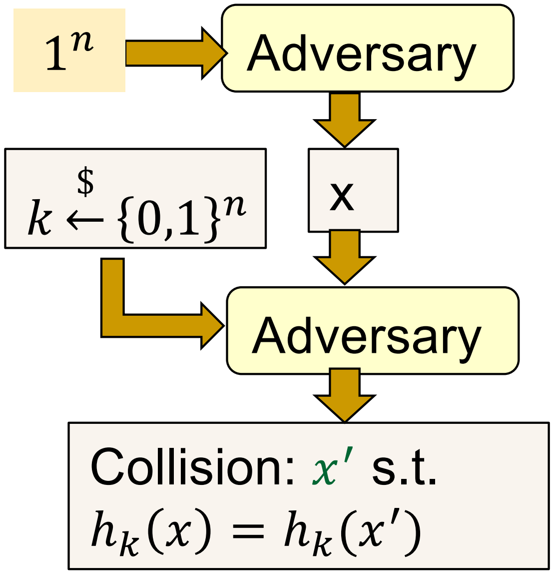
Range $\{0,1\}^n$

$h_k(x) = h_k(x')$

Adversary has to select target **before** knowing key

$$\varepsilon^{TCR}_{h,\mathcal{A}}(n) \equiv \Pr_{k \leftarrow \{0,1\}^n}\left[\left\{\begin{array}{c} x \leftarrow A(1^n); \\ x' \leftarrow A(x,k) \end{array}\right\} s.t.\ (x \neq x') \wedge (h_k(x) = h_k(x')\right]$$

TCR and Birthday Paradox?



1^n

Adversary

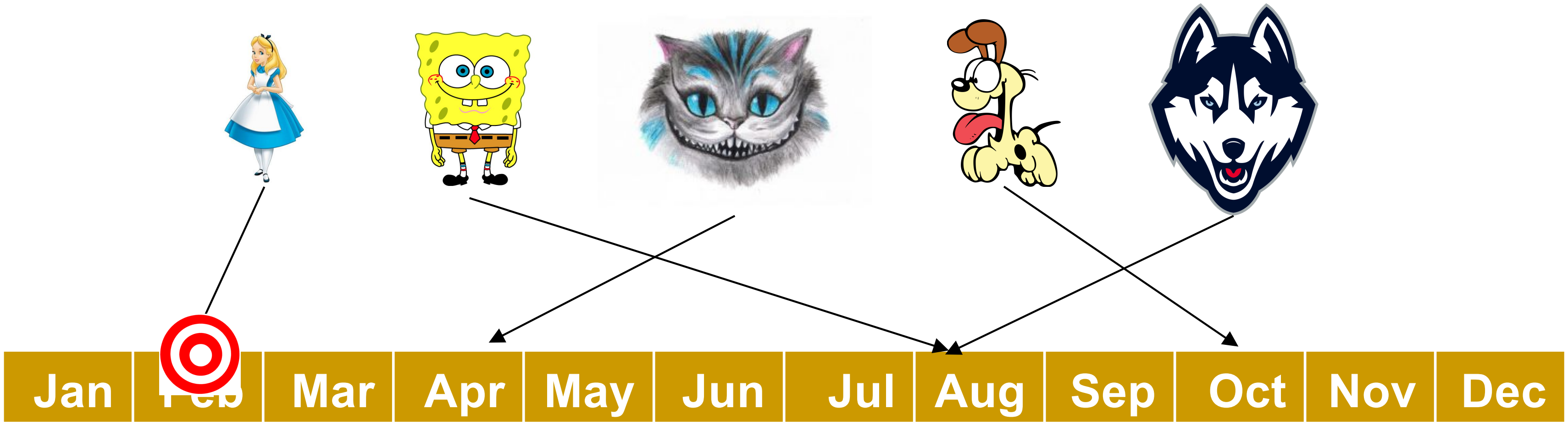
$k \xleftarrow{\$} \{0,1\}^n$

x

Adversary

Collision: x' s.t.
$h_k(x) = h_k(x')$

- **First:** adversary selects **x**
- Probability for NO birthday-collision **with x:**
  - Two persons: (364/365)
  - Three persons: (364/365)*(36**4**/365)
  - …
  - n persons: $\prod_{i=1}^{n-1} \frac{364}{365} = \left(\frac{364}{365}\right)^{n-1}$

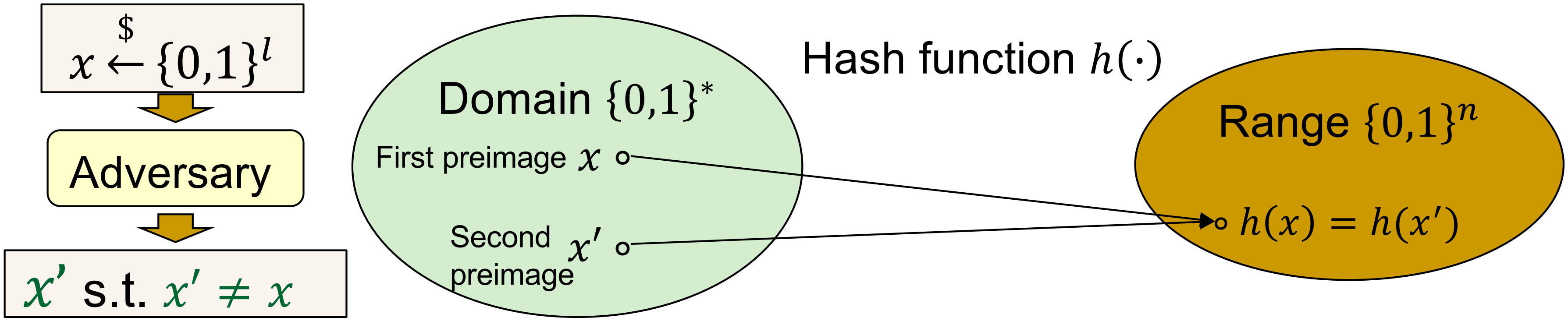


We (mostly) focus on keyless hash…

- Although there are no CRHFs
- And theory papers focus on keyed hash
- But…
  - It's a bit less complicated and easier to work with.
  - No need to consider both ACR and TCR
    - Why?
  - Modifying to ACR is quite trivial
    - Just make it keyed!
  - Usually used in practice: libraries, standards, …

2nd-Preimage-Resistant Hash (SPR)

- Hard to find collision with a <u>specific random x</u>.

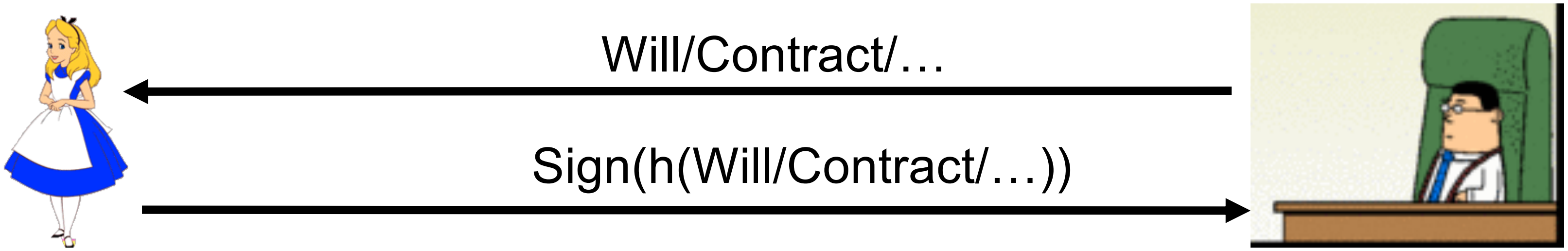


$$\varepsilon^{SPR}_{h,\mathcal{A}}(n) \equiv \Pr_{x \overset{\$}{\leftarrow} \{0,1\}^{A(1^n)}} [x' \leftarrow \mathcal{A}(x)\ s.t.\ x \neq x' \ \wedge\ h(x) = h(x')]$$

Use with care!
(think carefully about the security you want to achieve and see if SPR suffices)

CRHF/SPR vs. Applications

- CRHF secure for signing, SW-distribution
- How about SPR hash (weak-CRHF)?
  - SW-distribution? ***YES***
  - Hash-then-sign? ***NO***
- Why?
  - Attacker can't impact SW to be distributed
  - But… attacker may be able to impact signed msg!

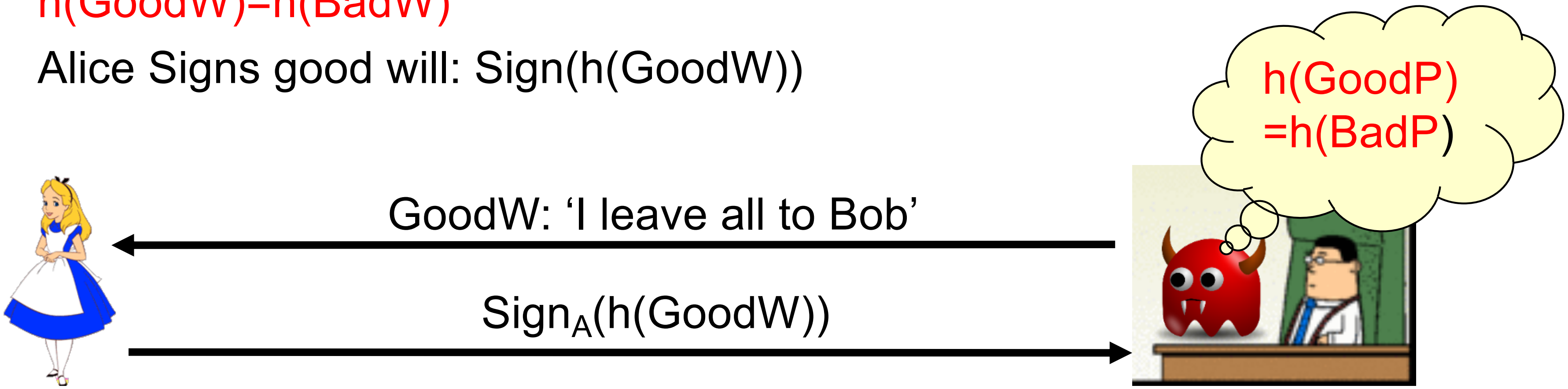


Will/Contract/…

Sign(h(Will/Contract/…))

SPR: Collisions to **Chosen** Messages

- Or: Alice and Mal, the corrupt lawyer
- Mal finds two `colliding wills’, GoodW and BadW:
  - GoodW: contents agreeable to Alice
  - h(GoodW)=h(BadW)
  - Alice Signs good will: Sign(h(GoodW))



h(GoodP)
=h(BadP)

GoodW: ‘I leave all to Bob’

Sign$_A$(h(GoodW))

- Later… Mal presents to the court:

BadW: ‘I leave all to Mal’, Sign$_A$(h(BadW))

$$$$

SPR: collisions to **chosen** message

- Or: Alice and Mal, the corrupt lawyer
- Mal finds two `colliding wills', GoodW and BadW:
  - GoodW: contents agreeable to Alice

Is such attack realistic?
Or SPR is enough ‘in practice’?

P)
P)

BadW: ‘I leave all to Mal’, $Sign_A(h(BadW))$

$$$$

SPR & Chosen-prefix vulnerability

- Chosen-prefix vulnerability :
  - Mal selects `prefix string’ p
  - Efficient alg finds :

    $x \neq x'$ s.t. $h(p||x) = h(p||x')$
  - Or, also for <u>any</u> suffix: $(\forall s)h(p||x||s) = h(p||x'||s)$
- Hash may be SPR yet allow chosen-prefix attacks
- Such attacks found for several proposed, standard cryptographic hash function, e.g., MD5 and SHA1
- We show chosen prefix attack on HtS
  - <u>Example of possible attack on HtS with SPR</u>

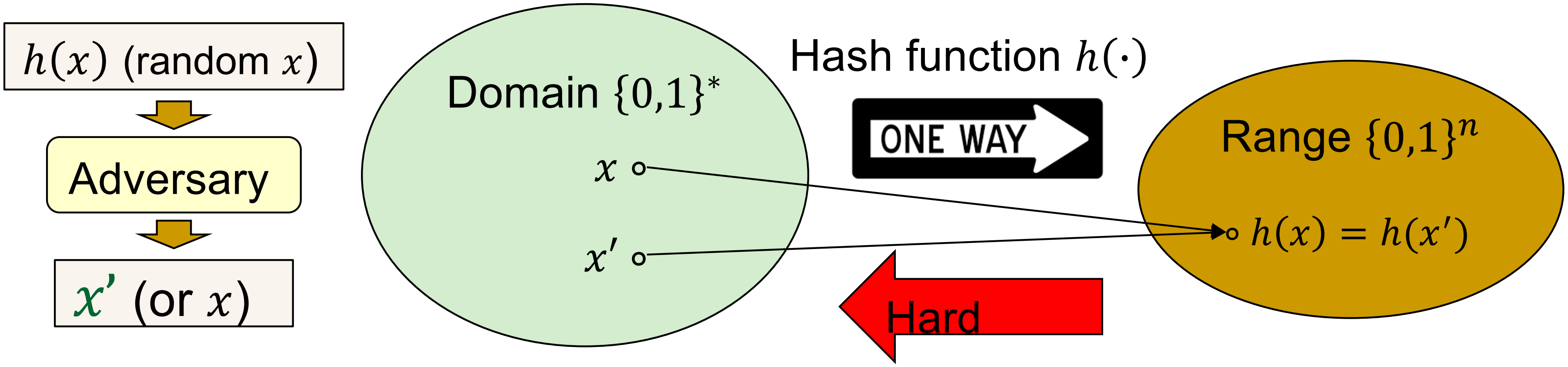
Chosen-prefix Attack

- Let $x < x'$ be collision for prefix: $p=$`Pay Mal $'
- Mal tricks Alice into signing him an IOU for $$x$
- Alice signs, sends $s=S_s^h(m)$ where $m=$ `Pay Mal $'||$x$
  - $S_s^h(m)=S_s(h(p||x))=S_s(h(p||x'))=S_s^h(m')$
  - $m'=$ `Pay Mal $'||$x'$
- Mal sends s, m' to Alice's bank
  - Bank validates $\text{“Ok”}=Verify_{Alice.v}(m',s)$
- Bank gives $$x'$ of Alice to Mal!!
- This attack is simplified:
  - Mal has to find `good' collision (high profit, convince Alice to sign)
  - People sign (PDF) files, not plain text…
- In reality, attacker also chooses suffix ➔ stronger attack

Examples

- On the whiteboard.

One-Way Function (OWF)



$h(x)$ (random x)

Adversary

x' (or x)

Domain $\{0,1\}^*$

x

x'

Hash function $h(\cdot)$

ONE WAY

Hard

Range $\{0,1\}^n$

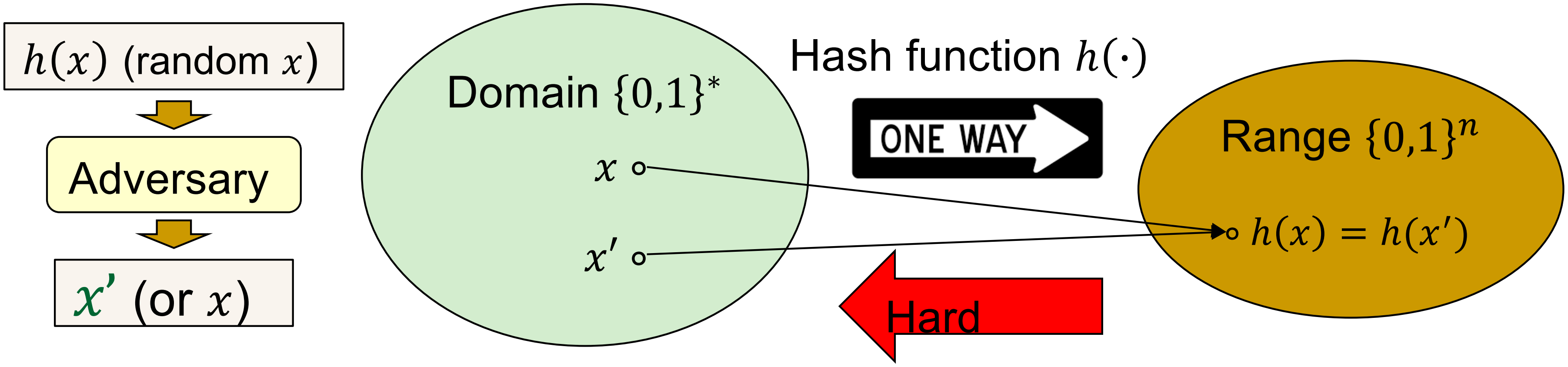
$h(x) = h(x')$

- **One-way function** or **first preimage resistance**: given $h(x)$ for random x, it is hard to find x, or any x' s.t. $h(x')=h(x)$

Compare to:

- Collision-Resistance (CR): hard to find collision, i.e., any (x, x') s.t. $h(x')=h(x), x \neq x'$
- Second-preimage resistance (SPR): hard to find collision with random x, i.e., x' s.t. $h(x')=h(x), x \neq x'$

Application: One-time Password Authentication



$h(x)$ (random x)

Adversary

x' (or x)

Domain $\{0,1\}^*$

x

x'

Hash function $h(\cdot)$

ONE WAY

Hard

Range $\{0,1\}^n$

$h(x) = h(x')$

- **One-time password authentication:**
  - Select random x : ‘one-time password’ (keep secret!)
  - Validate using non-secret ‘one-time validation token’: $h(x)$
- Extend to one-time public-key signatures.
  - Will be covered later when we study digital signatures.

How about a one-time password chain?

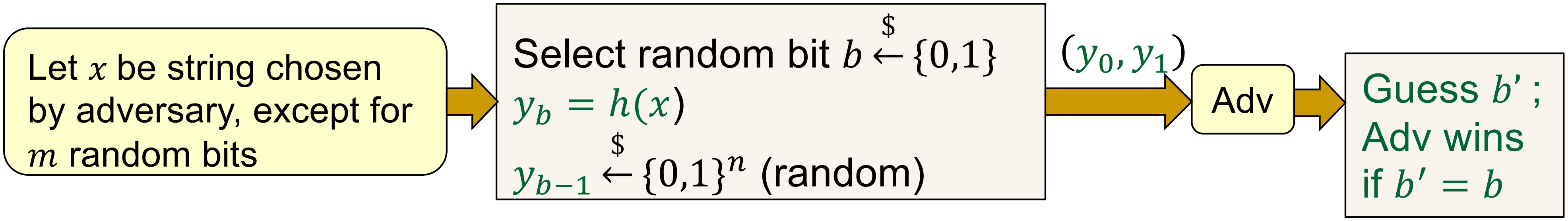
Not an Application: One-time Password Chain

- Alice computes a hash chain instead of one hash:
  - Select random x_0 then compute a chain of length l of hashes: $x_{i+1} = h(x_i)$
  - This allows Alice to authenticate l times instead of one.
    - Alice gives the server x_l then each time she wants to authenticate she sends x_{i-1}
    - The server can check by verifying that $x_i = h(x_{i-1})$
- A one-way function property alone may not sufficient, h has also to be a permutation.
  - x_i need to be uniformly distributed.

Example

- Let $h(x)$ be a OWF, construct $g(x)$ as:
    - $g(x) = 0^{2n}$ if x is a multiple of 2^n
    - $g(x) = h(x) \,||\, 0^n$ otherwise
- $g(x)$ is a OWF.
    - Why?
- But $f(x) = g(g(x))$ is not a OWF.
    - Why?
- And recall that a one time password chain is a nested calls of the hash function.
    - So $g(x)$ cannot be used to construct such a chain.
    - Why?

Randomness Extraction



Let x be string chosen by adversary, except for m random bits

Select random bit $b \overset{\$}{\leftarrow} \{0,1\}$
$y_b = h(x)$
$y_{b-1} \overset{\$}{\leftarrow} \{0,1\}^n$ (random)

(y_0, y_1)

Adv

Guess b' ; Adv wins if $b' = b$

- 'If input is sufficiently random, then output is random'
- Multiple `sufficiently random' models
- **Randomness extraction:** if any m input bits are random ➔ all n output bits are pseudorandom
  - For sufficiently large m
  - **Pseudorandom:** it is not computationally-feasible to distinguish between these bits and truly random bits
- How to model random extraction? Two models are discussed next!

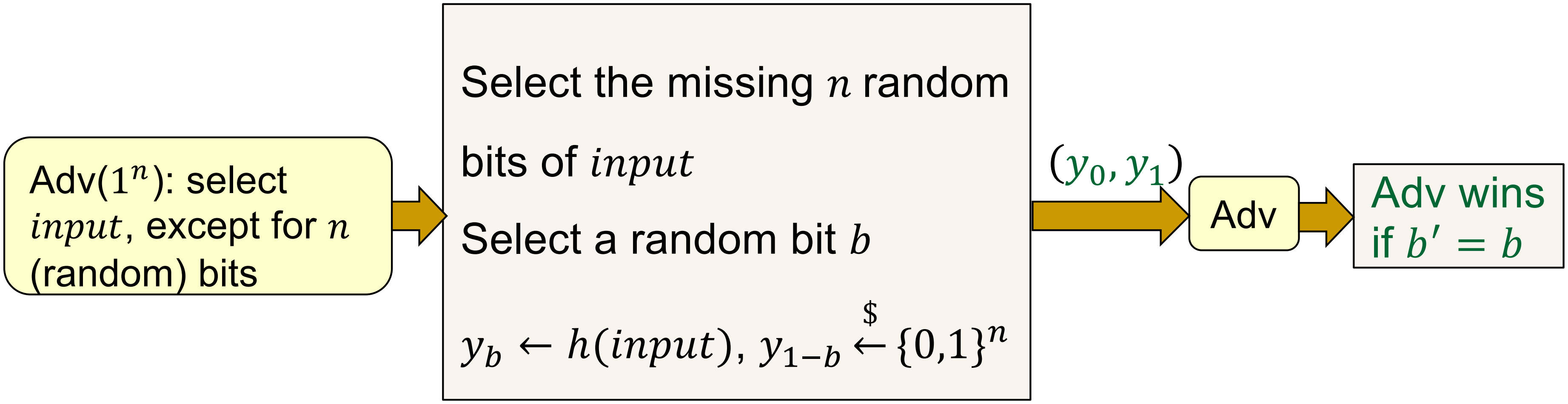
Von Neuman's Randomness Extractor

- Assume each bit is result of flip of coin with fixed bias
  - The bit 1 is produced with probability *p* and the bit 0 is produced with a probability *1 – p*
  - Coin tosses are independent.
- Von Neuman's solution:
  - Arrange input in pairs of bits: $\{(x_i, y_i)\}$
  - Remove pairs where bits are the same, so now $x_i \neq y_i$
  - Output x_i
- If assumption holds (independent biased coin flips) – output is uniform !
  - Bit 0 or 1 is produced with probability exactly ½

Bitwise Randomness Extraction

- ‘If input is sufficiently random, then output is random’
- Simple model: if any n input bits are random,

  ➔ all n output bits are pseudorandom
  - For sufficiently large n
- Simplified process:



Adv(1^n): select $input$, except for n (random) bits

Select the missing n random bits of $input$

Select a random bit b

$y_b \leftarrow h(input)$, $y_{1-b} \overset{\$}{\leftarrow} \{0,1\}^n$

(y_0, y_1)

Adv

Adv wins if $b' = b$

Random Oracle Model (ROM)

- Use a fixed, keyless hash function h
- Analyse as if hash $h()$ is a *random function*
  - An invalid assumption: $h()$ is fixed!
  - Whenever $h()$ is used, use oracle (black box) for random function
- Good for screening insecure solutions
  - Random oracle security → many attacks fail
- In practice: assume random oracle and use a standard hash function
  - It was shown that in some cases the construction will become insecure.
- Better to have security with standard assumption than the non-standard ROM.

Exercise

- Let h_1, h_2 be both CRHF and OWF
- Use them to construct:
  - h_{CRHF} - CRHF but not OWF
  - h_{OWF} - OWF but not CRHF
- One possible solution:
  - $h_{CRHF}(m) = \{1||m \text{ if } |m|=n,\ \ 0||h_1(m) \text{ otherwise } \}$
  - $h_{OWF}(m) = \begin{cases} h_1(m)\ if\ |m| = n \\ h_1(m_{1..n} \oplus h_2(m'))\ if\ m = m_{1..n}||m' \end{cases}$

Covered Material From the Textbook

- Chapter 4
  - Sections 4.1, 4.2 (except 4.2.6), 4.3, 4.4 (except 4.4.2), 4.5 (except 4.5.3).

Thank You!